Inner crust of neutron stars with mass-fitted Skyrme interaction

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Introduction

The neutron stars (NSs) observed with pulses of radio waves and other electromagnetic radiations are known as pulsars [1]. An irregularity in the rotational frequency, known as "glitch" has been observed in Radio Pulsars [2] and more recently in anomalous X- ray pulsars [3]. Anderson and Itoh suggested that glitches are triggered by the sudden unpinning of superfluid vortices in neutron star crust [4]. The crust is believed to be the reservoir of angular momentum which results in the glitch spin up [5]. Hence glitch is related to the crustal fraction of the moment of inertia denoted by ΔI/I. In the present work the Skyrme force SLy9 [6] is used. The core-crust transition density and the corresponding pressure are determined by considering the β-stability condition of nuclear matter with respect to the thermodynamic stability conditions [7]. The Tolman-Oppenheimer-Volkoff equation (TOV) is used to derive the mass and radius relationship of NS. The crustal fraction of the total moment of inertia is determined by using core-crust transition density (ρ_t) and the corresponding pressure $P(\rho_t)$ which allows us to limit the radius of Vela pulsar.

Formalism

The transition density (ρ_t) is calculated from the onset of instability of the uniform liquid against small amplitude density fluctuation due to clusterization. This is examined by analyzing the thermodynamical stability conditions in the β-equilibrated dense n+p+e+μ matter which is given by [8,9],

$$V_{thermal} = 2\rho\left(\frac{\partial e(\rho,Y_p)}{\partial \rho}\right) + \rho^2\left(\frac{\partial^2 e(\rho,Y_p)}{\partial \rho^2}\right) - \frac{\left(\rho\frac{\partial^2 e(\rho,Y_p)}{\partial\rho\partial Y_p}\right)^2}{\left(\frac{\partial^2 e(\rho,Y_p)}{\partial {Y_p}^2}\right)} > 0 \qquad ...(1)$$

where, $e(\rho,Y_p)$ is the energy per baryon.

The crustal fraction of the moment of inertia ΔI/I contains the mass M and radius R of the NS and is given by the following approximate expression[10],

$$\frac{\Delta I}{I} \approx \frac{28\pi P(\rho_t)R^3}{3Mc^2}\left(\frac{1-1.67\xi-0.6\xi}{\xi}\right) \times \left(1 + \frac{2P(\rho_t)}{\rho_t mc^2}\frac{(1+7\xi)(1-2\xi)}{\xi^2}\right)^{-1} \qquad ...(2)$$

where $\xi = \frac{GM}{RC^2}$, G is the gravitational costant and c is the velocity of light.

The skyrme interaction used in the present study is given by [11],

$$V(\rho_n,\rho_p) = \frac{t_0}{4}[(2+x_0)\rho^2 - (1+2x_0)(\rho_n^2 + \rho_p^2)] + \frac{t_3}{24}\rho^\gamma[(2+x_3)\rho^2 - (1+2x_3)(\rho_n^2 + \rho_p^2)] + \frac{1}{8}\rho^\gamma[t_2(1+2x_2) - t_1(1+2x_1)](\tau_n\rho_n + \tau_p\rho_p) + \frac{1}{8}\rho^\gamma[t_1(2+x_1) + t_2(2+x_2)]\tau\rho \qquad ....(3)$$

where, $\rho_n+\rho_p= \rho$= total nucleon density, $\tau = \tau_p + \tau_n$, $\tau_i= \frac{3}{5}k_i^2\rho_i$, and i= n, p. The Skyrme interaction altogether contains nine parameters, namely, γ, t_0, x_0, t_1, x_1, t_2, x_2, t_3, x_3.

Results and Discussion

The transition density and pressure, required for the calculation of moment of inertia apart from the bulk properties of the NS, is calculated in the thermodynamical spinoidal formulation. The mass-radius relation in Vela pulsar is obtained by calculating the crustal fraction of the moment of inertia in a NS. These calculations are made using the Skyrme Sly9 interaction. Table-1 represents the straight line equation for the radius R of the Vela pulsar as a function of its mass M for the present EOS. The variation of mass with the central density is calculated for a slowly rotating neutron star for the present EOS using TOV equation which is shown in Fig. 1. Pulsars with masses 1.88 $M_\odot$ or less have a crustal fraction of the total moment of inertia greater than 0.014, i.e. $\Delta I/I > 0.014$. In the present equation of state, the allowed masses and radii of the Vela pulsar glitches will be $R \geq 8.39+3.78M/M_\odot$ and $R \geq 3.75+3.78M/M_\odot$ for $\Delta I/I > 0.07$ and $\Delta I/I > 0.014$, respectively, which is shown in Fig. 2. Our calculation would mean that the crustal fraction of the moment of inertia can be ~ 6.1% due to crustal entrainment. The calculations suggest that without entrainment, the crust is enough to explain the Vela glitch data and with entrainment, the crust is not enough since the mass of Vela pulsar would be below 1$M_\odot$ (fig. 2), in accordance with other studies [12-14]

Table 1. The transition density ρ_t, the corresponding pressure P_t and Vela Pulsar Radius constraints are given in fm^{-3}, $MeVfm^{-3}$ and km respectively.

Interaction	ρ_t (fm^{-3})	P_t (MeV fm^{-3})	$Y_p(\rho_t)$	Vela Pulsar Radius constraint (Km)
SLy9	0.0803	0.3591	0.036	$R \geq 3.75 + 3.78M/M_\odot$
KDE0v1 [15]	0.0904	0.5013	0.041	$R \geq 3.69 + 3.44M/M_\odot$

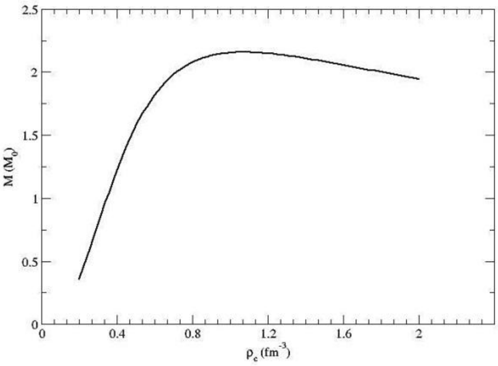

Fig. 1 Variation of mass (M) with central density (ρ_c) for slowly rotating NSs for the present nuclear EOS.

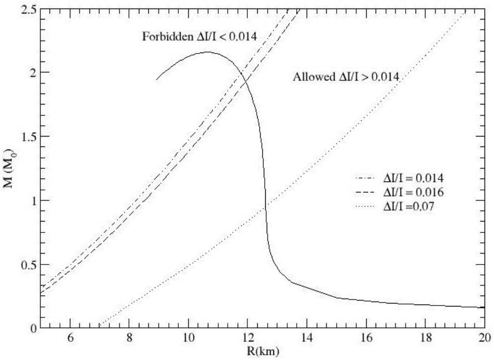

Fig. 2 The mass-radius relation of slowly rotating NSs for the present nuclear EOS. The constraint of $\Delta I/I > 1.4\%$ (1.6%, 7%) for the Vela pulsar implies that to the right of the line defined by $\Delta I/I = 0.014$ (0.016, 0.07) (for $\rho_t = 0.08027\ fm^{-3}$, $P_t = 0.3591 MeVfm^{-3}$), allowed masses and radii lie.

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