Study of the mass spectra of mesons using asymptotic iteration method

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Introduction

The spectroscopic study of mesons provides a good description of their internal structure like mass spectra, decay constant, radiative transition rates etc. Basically mesons are formed from quarks and antiquarks. The mass spectra of mesons are studied relativistically employing quantum chromodynamics using Cornell potential, but it is complex and computationally expansive. In non-relativistic framework, mass spectra of mesons can be obtained using a potential model approach.

In the potential model approach, problem can be solved mathematically using the Schrodinger equation for interaction potential exists among particles of the system. Here, in this work, the interaction potential between quarks is considered as

$$V(r) = ar^2 + br - \frac{c}{r} + \frac{d}{r^2}, \qquad (1)$$

where a, b, c and d are the potential parameters whose values are to be determined later. This potential have also application in quantum dot systems [1].

Formalism

The 3-dimensional Schrodinger equation ($\hbar = 1$) for the interaction potential (1) is written as

$$\left[\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{l(l+1)}{r^2} + 2\mu\begin{pmatrix} E - ar^2 - br \\ + \frac{c}{r} - \frac{d}{r^2} \end{pmatrix}\right]\psi(r) = 0 \qquad (2)$$

where $l = 0,1,2..........$ is angular momentum quantum number, μ is reduced mass of the system, r is inter quark separation and E denotes the energy eigenvalues of the considered system.

Now inserting $\psi(r) = \frac{1}{r}R(r)$ in equation (2), we get

$$\left[\frac{d^2}{dr^2} + \frac{\eta^2 - \frac{1}{4}}{r^2} + \begin{pmatrix} \varepsilon - a_1 r^2 - b_1 r \\ + \frac{c_1}{r} \end{pmatrix}\right]R(r) = 0 \qquad (3)$$

where $\eta = L + \frac{1}{2}$, $L = \frac{-1 + \sqrt{(2l+1)^2 + 4d_1}}{2}$

$\varepsilon = 2\mu E$, $a_1 = 2\mu a$, $b_1 = 2\mu b$, $c_1 = 2\mu c$ and $d_1 = 2\mu d$.

This equation (3) is similar to equation (11) of ref [2]. So by adopting the same methodology of asymptotic iteration method of ref [2], we get the mass spectra equation as

$$M_{nl} = m_1 + m_2 + \sqrt{\frac{a}{2\mu}}\left(2n + 2 + \sqrt{\begin{matrix}(2l+1)^2 \\ + 8\mu d\end{matrix}}\right) + \left[\frac{4n - 1 - \sqrt{(2l+1)^2 + 8\mu d}}{1 + \sqrt{(2l+1)^2 + 8\mu d}}\right]\frac{b^2}{4a} \qquad (4)$$

and the restriction on potential parameters as

$$c = \left[2n + 1 + \sqrt{(2l+1)^2 + 8\mu d}\right]\frac{b}{2\sqrt{2\mu a}} \qquad (5)$$

Here, we have taken the quark masses as, $m_b = 4.823$ GeV, $m_s = 0.419$ GeV and $m_u = m_d = 0.220$ GeV. The values of potential parameters a and b are obtained by using experimental data for states 1S and 1P after fixing parameter d in equation (4). The parameter c is then computed using restriction, equation (5). The mass of other states 2S, 2P and 1D are calculated by putting the values of a, b and d in equation (4). The complete mass spectra of $b\bar{s}$ and $b\bar{q}$ mesons are tabulated in tables 1 & 2 and the shape of interaction potential (1) is presented in figures 1 and 2.

Table 1: Mass spectra of $b\bar{s}$ meson for potential parameter values $a = 0.0372$, $b = 0.2796$ and $d = 0.1232$ in GeV/C^2.

State	Present Work	Exp. [3,4]	Relativistic [5]
1S	5.415	5.415*	5.450
2S	6.819	-	6.012
1P	5.830	5.830*	5.857
2P	6.786	-	6.279
1D	6.264	-	6.182

Table 2: Mass spectra of $b\bar{q}$ $(q = u, d)$ meson for potential parameter values $a = 0.0178$, $b = 0.1589$ and $d = 0.1232$ in GeV/C^2.

State	Present work	Exp. [3,4]	Relativistic [5]
1S	5.325	5.325*	5.371
2S	6.413	-	5.933
1P	5.723	5.723*	5.777
2P	6.486	-	6.197
1D	6.131	-	6.110

Conclusion

In the present work, we have computed mass spectra of $b\bar{s}$ and $b\bar{q}$ mesons within the framework of interaction potential (1) invoking asymptotic iteration method. The results given in tables 1 and 2 are in excellent agreement with the experimental data and results of relativistic quark model [5].

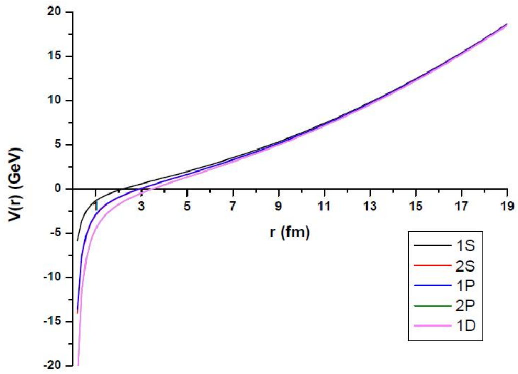

Fig. 1 Variation of interaction potential (1) with inter quark separation for $b\bar{s}$.

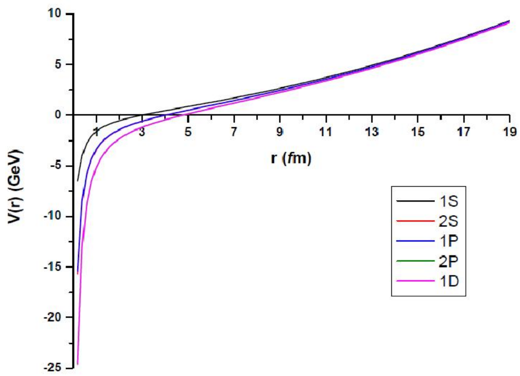

Fig. 2 Variation of interaction potential (1) with inter quark separation for $b\bar{q}$.

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References

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