Spectral function of $\rho$ at finite temperature in arbitrary external magnetic field

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Introduction

The study of “strongly” interacting matter in presence of high external magnetic field has been a significant topic of research over a decade due to the possibility of exotic phenomena and effects like Chiral Magnetic Effect (CME), Magnetic Catalysis (MC), Inverse Magnetic Catalysis (IMC), vacuum superconductivity etc. Such situation can be physically realized in non-central Heavy Ion Collisions (HIC) and inside a magnetar.

In this work, we are interested to study the spectral modification of $\rho$ meson under external magnetic field as well as at finite temperature. The spectral function of $\rho$ is an essential input to the calculation of low invariant mass dilepton spectra in HIC experiments.

Formalism

In order to calculate the 1-loop self energy of $\rho$, the following effective Lagrangian is taken,

$$\mathcal{L}_{\text{int}} = -g_{\pi \pi} \partial_{\mu} \rho \cdot \partial^{\mu} \pi \times \partial^{\mu} \pi,$$

with the coupling constant $g_{\pi \pi} = 20.72$ GeV$^{-2}$. Using $\mathcal{L}_{\text{int}}$, the 1-loop vacuum self energies of $\rho^0$ and $\rho^\pm$ can be written as,

$$\langle \Pi_{\mu \nu}^{\rho^0}(q) \rangle_v = i \int \frac{d^4k}{(2\pi)^4} N^{\mu\nu}(q,k)\Delta_{0}(k)\Delta_{\pm}(p)$$

$$\langle \Pi_{\mu \nu}^{\rho^\pm}(q) \rangle_v = i \int \frac{d^4k}{(2\pi)^4} N^{\mu\nu}(q,k)\Delta_{0}(k)\Delta_{\pm}(p)$$

where, $\Delta_{0}(k) = \left( \frac{1}{k^2 - m_0^2 + i\epsilon} \right)$ and $\Delta_{\pm}(k) = \left( \frac{1}{k^2 - m_{\pm}^2 + i\epsilon} \right)$ are the vacuum Feynman propagators of $\pi^0$ and $\pi^\pm$ with masses $m_0$ and $m_{\pm}$ respectively and $N^{\mu\nu}(q,k)$ contains factors coming from interaction vertices and is given by,

$$N^{\mu\nu}(q,k) = g_{\pi \pi}^2 \left[ q^\mu k^\nu + (q.k)^2 q^\mu q^\nu - q^2(q.k)(q^\mu k^\nu + k^\mu q^\nu) \right].$$

To calculate the self energy at finite temperature and external magnetic field, we follow standard methods of real time formalism of thermal field theory in which the self energy becomes $2 \times 2$ matrix. This thermal self energy matrix can be diagonalized to obtain analytic self energy function which is related to the 11 component of the said matrix.

The 11-component of the $\rho$ self energy at finite temperature and external magnetic field is obtained by replacing the $\pi^0$ and $\pi^\pm$ propagators in Eq. (1) by the following,

$$D_0^{11}(k) = \Delta_0(k) + 2i\eta^k \text{Im} \Delta_0(k)$$

$$D_{\pm}^{11}(k) = \Delta_{\pm}(k) + 2i\eta^k \text{Im} \Delta_{\pm}(k)$$

where

$$\Delta_{\pm}(k) = -\sum_{l=0}^{\infty} \frac{2(-1)^l L_l(-2k_\perp^2/\epsilon B)e^{k_\perp^2/\epsilon B}}{k^2 - m_\pm^2 - (2l + 1)\epsilon B + i\epsilon}$$

is the Schwinger proper-time propagator for a charged scalar field and $\eta^k = \left[ e^{k.\nu/T} - 1 \right]^{-1}$ is the Bose-Einstein distribution function of pions with $u^\mu$ being the four velocity of the...
medium. The magnetic field is taken along +ve z-direction. Any 4-vector is decomposed into $k = (k_\parallel + k_\perp)$ with $k_\parallel^\mu \equiv (k^0, 0, 0, k_z)$ and $k_\perp^\mu \equiv (0, k_x, k_y, 0)$. The analytic self energy functions are related to the 11-component of self energy matrix by the following relations,

$$\text{Re} \bar{\Pi}_{\mu\nu}(q) = \text{Re} \Pi_{11\mu\nu}(q)$$

$$\text{Im} \bar{\Pi}_{\mu\nu}(q) = \epsilon(q^0) \tanh \left( \frac{q^0}{2T} \right) \text{Im} \Pi_{11\mu\nu}.$$  

The explicit form of analytic self energy functions may be found in Ref. [1]. The spectral function of $\rho$ energy in the rest frame of $\rho$ meson are evaluated at finite temperature under external magnetic field of arbitrary magnitude. No approximations are made on the strength of $eB$ and the calculation incorporates infinite number of Landau levels. Analytic structure of the imaginary part of the self energy is found to be different for $eB$ and $T$ dependent medium contribution. It is shown in Ref. [1], that the former term dominates over the later and is usually ignored in the literature. The imaginary part of the self energy is responsible for different physical processes like decay or scattering in respective kinematic domains and they correspond to branch cuts of the self energy function in the complex $q^0$ plane. The detail study of this analytic structure may be found in Ref. [1], where it is shown the magnetic field makes the Landau structure to loose its Breit-Wigner shape.

**Results and Discussions**

In figure Fig. 1(a), the spectral function of $\rho^0$ is shown. The spike like structure appears there due to “threshold singularity” in each Landau levels. With the increase of $eB$ the threshold of the spectral function moves towards higher $q^0$ and eventually it crosses the physical $\rho$ mass pole causing the spectral function to loose its Breit-Wigner shape. This may be termed as “melting” of $\rho^0$ at high $eB$. Corresponding results for $\rho^\pm$ are shown in Fig. 1(b), where the $\rho^\pm$ width decreases with the increase of $eB$ making it more stable.

In summary, the spectral functions of $\rho$ meson are evaluated at finite temperature under external magnetic field of arbitrary magnitude. No approximations are made on the strength of $eB$ and the calculation incorporates infinite number of Landau levels. Analytic structure of the imaginary part of the self energy is found to be different for $\rho^0$ and $\rho^\pm$ as well as from the $eB = 0$ case. Landau cuts appear in presence of $eB$ which is absent in zero magnetic field.

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**References**