Lepton-Proton Bremsstrahlung Distribution Function in Low-Energy Effective Field Theory

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Introduction

Although proton has been investigated for more then one hundred years, recent findings indicate that its actual radius is still a puzzle. Recently, a series of precise experiments of the proton radius have produced results [1] which are in strong disagreement casting doubt on our knowledge of one of the protons fundamental properties and our understanding of the underlying physics.

The proton radius puzzle is being addressed by a number of new experiments, the Muon proton Scattering Experiment (MUSE) at PSI being one of them, that proposes to measure the elastic lepton-proton (lp) scattering cross section at very low momentum transfer. In such an experiment corrections must be made for the fact that leptons can lose energy due to radiative effect such as bremsstrahlung. At low energies (below the chiral scale $\Lambda_\chi \sim 1$ GeV) , which includes the kinematic region of interest for the MUSE experiment, the dynamics of proton is governed by chiral symmetry. The *Chiral Perturbation Theory* (χPT), is a low-energy effective theory which incorporate the chiral symmetry and its breaking patterns and provide a systematic framework to study hadronic observable at low energies. In χPT the proton is effectively treated as an elementary particle and the evaluation of an observable follows well-defined chiral power counting rules for generating the dominant leading order (LO) contributions, as well as higher order corrections in a perturbative expansion scheme. Here we present a LO calculation of the bremsstrahlung process in lp scattering based on the heavy baryon formalism (HBχPT), in which the proton being heavy $m_p \sim \Lambda_\chi$ is considered effectively static.

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HBChPT formalism

We work in flavor $SU(2)$ version of χPT i.e., in the isospin limit $m_u = m_d$. The pion fields are collected in the matrix, $u(x) = exp[i\tau.\pi(x)/2f_\pi]$ where $f_\pi = 93$ MeV, is the pion decay constant. The lepton mass m_l is included in all our expressions systematically. The four momentum of the heavy nucleon is written as $P_\mu = mv_\mu + l_\mu$ with v_μ the velocity four-vector and l_μ is small residual momentum which satisfy the conditions $v^2 = 1$ and $v.l = -\frac{l^2}{2m_p} \approx 0$ at LO.

The leading order heavy baryon Lagrangian which we only need for our calculation is

$$\mathcal{L}^1_{\pi N} = \bar{H}(iv.D + g_A S.u)H \qquad (1)$$

where $D_\mu = \partial_\mu + \Gamma_\mu$ with $\Gamma_\mu = \frac{1}{2}[u^\dagger(\partial_\mu - ir_\mu)u + u(\partial_\mu - il_\mu)u^\dagger]$. H denotes the heavy nucleon field, g_A is axial vector coupling constant, S_μ is the covariant spin operator, r_μ and l_μ are the external source fields. For interaction with external electromagnet field, $r_\mu = l_\mu = -e\frac{\tau_3}{2}\mathcal{A}_\mu$ and $v^{(s)}_\mu = -\frac{e}{2}\mathcal{A}_\mu$. We choose to work in in the Coulomb gauge, $\epsilon.v = 0$, which means that the photon does not couple to the proton at LO. Consequently the Feynman diagrams A and B in Fig.1 are evaluated to obtain our LO result. The incident and scattered lepton four-momenta are denoted as $p = (E_l, \vec{p})$ and $p' = (E'_l, \vec{p}\,')$, respectively and $k = (E_\gamma, \vec{k})$ denote the outgoing photon 4-momentum while θ is the lepton scattering angle.

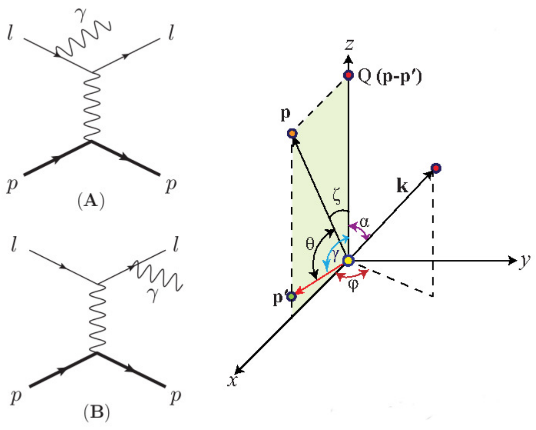

FIG. 1: Feynman diagrams contributing to leading order bremsstrahlung process(L.H.S) and reference coordinate system used in our calculation(R.H.S)

Preliminary Result

In this section we will report some preliminary results of our calculation. We define our coordinate system as shown in Fig.1. In this coordinate system we can readily perform the integration over photon azimuthal angle ϕ. We integrate over outgoing proton three momentum and photon energy to obtain the differential cross section $\frac{d\sigma}{d|\vec{p}\,'|\, d\Omega'\, d\cos\alpha}$. In Fig.2 we plot our cross section with $\cos\alpha$. From the electron plot we see that the cross section is sharply peaked along the direction of either the incident or the scattered electron which precisely in agreement with the standard peaking approximation [2] applied to radiative corrections for electron provided that p' is close to p. However, for large differences between p and p' such an approximation tends to fail. On the other hand, for muon we hardly see any peaks. This suggests that lepton mass crucially influence the bremsstrahlung photon's angular distribution and peaking approximation is questionable when kinematics typically involve low momentum transfer comparable to masses of the incident projectiles, such as the muon. This is a vital fact that needs to be incorporated in the lp scattering data analysis for an accurate extraction of the proton radius in future analysis.

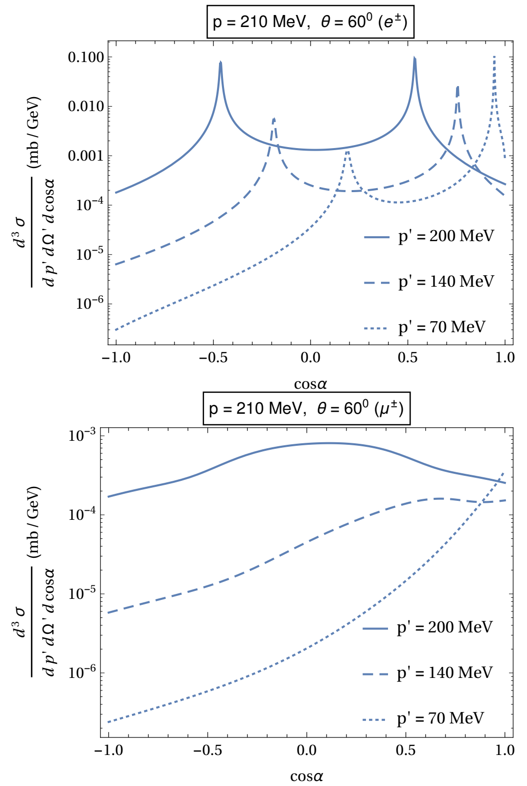

FIG. 2: Angular distributions of the cosine of the photon bremsstrahlung polar angle, $\cos\alpha$, for fixed $|\vec{p}|$ and θ and three different $|\vec{p'}|$ both for electron(e) and muon(μ). For the sake of comparison between the plots, a logarithmic ordinate scale is chosen.

More work is currently in progress to reduce systematic errors by determining the full bremsstrahlung distribution function incorporating the m_p^{-1} proton recoil corrections and a complete dynamic NLO calculation.

References

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