S-wave spectroscopy and Hyperfine splitting of B_c meson

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1. Introduction

B_c meson is the only heavy meson with two open flavours. This system is also interesting because they cannot annihilate into gluons [1]. The mass spectra and hyperfine splitting of the B_c meson are investigated in the Dirac framework with the help of linear + constant potential. The spin-spin interactions are also included in the calculation of the pseudoscalar and vector meson masses. Our computed result for the B_c meson are in very good agreement with experimental results as well as other available theoretical result. Decay properties are also interesting because it is expected that decay of B_c meson occur in to neutral meson. We hope our theoretical results are helpful for future experimental observations.

2. Theoretical Methodology

To study heavy quarkonia we have considered the confinement through a linear potential. The form of the model potential is expressed as,

$$V(r) = \frac{1}{2}(1+\gamma_0)(\lambda r^{1.0} + V_0) \quad (1)$$

Where λ is the strength of the confinement part of the potential. V_0 is a constant negative potential depth.

For the mass spectroscopy we compute the confining energy of the quark and anti quark by solving the Dirac equation. The solution of Dirac equation can be written as two component (positive and negative energies in the zeroth order) form as [2, 3],

$$\psi_{nlj}(r) = \begin{pmatrix} \psi^{(+)}_{nlj} \\ \psi^{(-)}_{nlj} \end{pmatrix} \quad (2)$$

where

$$\psi^{(+)}_{nlj}(\vec{r}) = N_{nlj} \begin{pmatrix} ig(r)/r \\ (\sigma.\hat{r})f(r)/r \end{pmatrix} \mathcal{Y}_{ljm}(\hat{r}) \quad (3)$$

$$\psi^{(-)}_{nlj}(\vec{r}) = N_{nlj} \begin{pmatrix} i(\sigma.\hat{r})f(r)/r \\ g(r)/r \end{pmatrix} (-1)^{j+m_j-l} \mathcal{Y}_{ljm}(\hat{r}) \quad (4)$$

and N_{nlj} is the overall normalization constant [2, 3]. The normalized.

The reduced radial part $g(r)$ and $f(r)$ of the Dirac spinor $\psi_{nlj}(r)$ are given by [2, 3],

$$\frac{d^2g(r)}{dr^2} + \left[(E_q+m_q)[E_q-m_q-V(r)] - \frac{\kappa(\kappa+1)}{r^2}\right] g(r) = 0 \quad (5)$$

and

$$\frac{d^2f(r)}{dr^2} + \left[(E_q+m_q)[E_q-m_q-V(r)] - \frac{\kappa(\kappa-1)}{r^2}\right] f(r) = 0 \quad (6)$$

On converting these equation into dimensionless form [3, 4] as,

$$\frac{d^2g(\rho)}{d\rho^2} + \left[\epsilon - \rho^{1.0} - \frac{\kappa(\kappa+1)}{\rho^2}\right] f(\rho) = 0 \quad (7)$$

$$\frac{d^2f(\rho)}{d\rho^2} + \left[\epsilon - \rho^{1.0} - \frac{\kappa(\kappa-1)}{\rho^2}\right] g(\rho) = 0 \quad (8)$$

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TABLE I: S-wave mass spectrum for B_c meson(in GeV).

nL	State	Present	[5]	[6]
1S	1^3S_1	6.331	6.333	6.331
	1^1S_0	6.270	6.272	6.275
2S	2^3S_1	6.874	6.882	6.890
	2^1S_0	6.825	6.842	6.852
3S	3^3S_1	7.249	7.258	7.268
	3^1S_0	7.211	7.226	-
4S	4^3S_1	7.604	7.609	-
	4^1S_0	7.582	7.585	-

TABLE II: The hyperfine splitting (in GeV) for S-wave for B_c meson(in GeV)

State (ns)	Hyperfine splitting	Present	[5]	[6]
1S	$\triangle M(1S)$	0.061	0.061	0.056
2S	$\triangle M(2S)$	0.049	0.040	0.038
3S	$\triangle M(3S)$	0.038	0.032	-
4S	$\triangle M(4S)$	0.022	0.024	-

where $\rho = \frac{r}{((E_q+m_q)\lambda)^{\frac{-1}{3}}}$ and $\epsilon = (E_q - m_q - V_0)(m_q + E_q)^{\frac{1}{3}}\lambda^{\frac{-2}{3}}$

Mass of particular Quark-Anti quark system can be written as [3],

$$M_{Q\bar{Q}} = E_Q + E_{\bar{Q}} - E_{cm} \quad (9)$$

here, E_{cm} in general can be state dependent which we absorb in our potential parameter V_0. Thus, making V_0 as state dependent. In our calculation we have also incorporated spin spin interaction for hyperfine splitting. the j-j coupling of confined one gluon exchange potential (COGEP) is given by, [3]

$$\langle V_{Q\bar{Q}}^{j_1 j_2} \rangle = \frac{\sigma \langle j_1 j_2 JM | \hat{j_1} . \hat{j_2} | j_1 j_2 JM \rangle}{(E_Q + m_Q)(E_{\bar{Q}} + m_{\bar{Q}})} \quad (10)$$

3. Mass spectra and hyperfine splitting in B_c meson

The computed S - wave and hyperfine splitting of B_c meson are tabulated in Table I and II.The hyperfine splitting for S-Wave is given by,

$$\triangle M_{hf}(nS) = M(n^3S_1) - M(n^1S_0) \quad (11)$$

Where, $M(n^3S_1)$ is mass of B_c^* state and $M(n^1S_0)$ is the mass of B_c meson.

4. Result and discussion

we have computed the mass spectrum of B_c meson in the relativistic Dirac formalism. We have compared our results on B_c^* and other radial excited states with available theoretical results. Available experimental results for 1^1S_0 is 6273.7 ± 1.3 ± 1.6 MeV [7] and for 2^1S_0 is 6842 ± 4 ± 5 MeV [8]. Our predicted results are in good agreement with these experimental results. The hyperfine splitting of nS states predicted by the model are also in very good agreement with other theoretical predictions of Ebert et al. based on cornell-like potential[5]. Further study on the decay properties will be presented.

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