S-wave spectroscopy and Hyperfine splitting of $B_c$ meson

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1. Introduction

$B_c$ meson is the only heavy meson with two open flavours. This system is also interesting because they cannot annihilate into gluons [1]. The mass spectra and hyperfine splitting of the $B_c$ meson are investigated in the Dirac framework with the help of linear + constant potential. The spin-spin interactions are also included in the calculation of the pseudoscalar and vector meson masses. Our computed result for the $B_c$ meson are in very good agreement with experimental results as well as other available theoretical result. Decay properties are also interesting because it is expected that decay of $B_c$ meson occur in to neutral meson. We hope our theoretical results are helpful for future experimental observations.

2. Theoretical Methodology

To study heavy quarkonia we have considered the confinement through a linear potential. The form of the model potential is expressed as,

$$V(r) = \frac{1}{2}(1 + \gamma_0)(\lambda r^{1.0} + V_0)$$

(1)

Where $\lambda$ is the strength of the confinement part of the potential. $V_0$ is a constant negative potential depth.

For the mass spectroscopy we compute the confining energy of the quark and anti quark by solving the Dirac equation. The solution of Dirac equation can be written as two component (positive and negative energies in the zeroth order) form as [2, 3],

$$\psi_{nlj}(r) = \begin{pmatrix} \psi_{nlj}^{(+)(r)} \\ \psi_{nlj}^{(-)(r)} \end{pmatrix}$$

(2)

where

$$\psi_{nlj}^{(+)(r)} = N_{nlj} \left( \frac{i g(r)/r}{\sigma.\vec{r}} \right) Y_{ljm}(\hat{r})$$

(3)

and $N_{nlj}$ is the overall normalization constant [2, 3]. The normalized.

$$\psi_{nlj}^{(-)(r)} = N_{nlj} \left( \frac{i (\sigma.\vec{r})f(r)/r}{g(r)/r} \right) \left( -1 \right)^l m_j - j m l Y_{ljm}(\hat{r})$$

(4)

and $N_{nlj}$ is the overall normalization constant [2, 3]. The normalized.

The reduced radial part $g(r)$ and $f(r)$ of the Dirac spinor $\psi_{nlj}(r)$ are given by [2, 3],

$$\frac{d^2 g(r)}{dr^2} + \left[ (E_q + m_q)[E_q - m_q - V(r)] - \frac{\kappa(\kappa + 1)}{r^2} \right] g(r) = 0$$

(5)

$$\frac{d^2 f(r)}{dr^2} + \left[ (E_q + m_q)[E_q - m_q - V(r)] - \frac{\kappa(\kappa - 1)}{r^2} \right] f(r) = 0$$

(6)

On converting these equation into dimensionless form [3, 4] as,

$$\frac{d^2 g(\rho)}{d\rho^2} + \left[ \epsilon - \rho^{1.0} - \frac{\kappa(\kappa + 1)}{\rho^2} \right] g(\rho) = 0$$

(7)

$$\frac{d^2 f(\rho)}{d\rho^2} + \left[ \epsilon - \rho^{1.0} - \frac{\kappa(\kappa - 1)}{\rho^2} \right] f(\rho) = 0$$

(8)

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TABLE I: S-wave mass spectrum for $B_c$ meson (in GeV).

<table>
<thead>
<tr>
<th>nL</th>
<th>State (ns)</th>
<th>Present</th>
<th>Refs</th>
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</thead>
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<tr>
<td>1S</td>
<td>$1^2S_1$</td>
<td>6.331</td>
<td>[5]</td>
</tr>
<tr>
<td></td>
<td>$1^1S_0$</td>
<td>6.270</td>
<td>[6]</td>
</tr>
<tr>
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<td></td>
<td>$2^1S_0$</td>
<td>6.825</td>
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<tr>
<td></td>
<td>$3^1S_0$</td>
<td>7.211</td>
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<tr>
<td>4S</td>
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<tr>
<td></td>
<td>$4^1S_0$</td>
<td>7.582</td>
<td></td>
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</table>

TABLE II: The hyperfine splitting (in GeV) for S-wave for $B_c$ meson (in GeV)

<table>
<thead>
<tr>
<th>State (ns)</th>
<th>Hyperfine splitting</th>
<th>Present</th>
<th>Refs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1S</td>
<td>$\Delta M(1S)$</td>
<td>0.061</td>
<td>[5]</td>
</tr>
<tr>
<td></td>
<td>$\Delta M(2S)$</td>
<td>0.049</td>
<td>[6]</td>
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<tr>
<td>3S</td>
<td>$\Delta M(3S)$</td>
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<tr>
<td></td>
<td>$\Delta M(4S)$</td>
<td>0.022</td>
<td></td>
</tr>
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</table>

where

$$\rho = \frac{r}{(E_q + m_q)\lambda}$$

and

$$\epsilon = (E_q - m_q - V_0)(m_q + E_q)^{\frac{1}{2}}\lambda$$

Mass of particular Quark-Anti quark system can be written as [3],

$$M_{QQ} = E_Q + E_Q - E_{cm}$$

Here, $E_{cm}$ in general can be state dependent which we absorb in our potential parameter $V_0$. Thus, making $V_0$ as state dependent. In our calculation we have also incorporated spin spin interaction for hyperfine splitting, the j-j coupling of confined one gluon exchange potential (COGEP) is given by, [3]

$$\langle V_{ij}^{ji}\rangle = \frac{\sigma(j_i j_j I M j_i j_j M)}{(E_Q + m_Q)(E_Q + m_Q)}$$

3. Mass spectra and hyperfine splitting in $B_c$ meson

The computed S-wave and hyperfine splitting of $B_c$ meson are tabulated in Table I and II. The hyperfine splitting for S-Wave is given by

$$\Delta M_{hf}(nS) = M(n^3S_1) - M(n^1S_0)$$

Where, $M(n^3S_1)$ is mass of $B_c^+$ state and $M(n^1S_0)$ is the mass of $B_c$ meson.

4. Result and discussion

we have computed the mass spectrum of $B_c$ meson in the relativistic Dirac formalism. We have compared our results on $1^1S_0$ and other radial excited states with available theoretical results. Available experimental results for $1^1S_0$ is $6273.7 \pm 1.3 \pm 1.6$ MeV [7] and for $2^1S_0$ is $6842 \pm 4 \pm 5$ MeV [8]. Our predicted results are in good agreement with these experimental results. The hyperfine splitting of nS states predicted by the model are also in very good agreement with other theoretical predictions of Ebert et al. based on cornell-like potential [5]. Further study on the decay properties will be presented.

Acknowledgments

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References