# Mass Spectrum of Orbitally Excited $B_s$ Mesons in a Non Relativistic Quark Model with Hulthen Potential

A P Monteiro[1],* Praveen P D'Souza [1] [2], and K. B. Vijaya Kumar [2]
[1]*P.G Department of Physics, St Philomena College, Darbe, Puttur-574202,INDIA and*
[2]*Department of Physics, Mangalore University,*
*Mangalagangothri P.O., Mangalore - 574199, INDIA*

### Introduction

Heavy light mesons composed of one heavy quark and one light quark,They are the only mesons containing quarks of the third generation. Understanding the B mesons will give a more complete understanding of excited mesons and will also help put the newly discovered excited charmed mesons into the larger context. In this respect, the physics of B mesons is complementary to that of the K mesons, which has contributed enormously to our understanding of elementary particles and their interactions. The rare decay of a neutral B meson to two oppositely charged kaons has been observed for the first time in LHCb experiment at CERN. B mesons are created when protons collide in the Large Hadron Collider and the observed decay happens to fewer than one in ten million B mesons. Despite many of the $B_s$ states should be accessible by the B-factories (CLEO,BaBar,Belle) and also by the proton antiproton colliders (CDF and D0), much of the b$s\bar{s}$ excitation spectrum remains to be observed. Only the ground S-wave spin singlet and spin-triplet states ($B_s and B_s^*$) and the orbitally excited $B_{s1}(5830)$ and $B_{s2}^*(5840)$ mesons are presently well established.

### Theoretical Background

The Hamiltonian employed in our model includes kinetic energy part, confinement potential and one gluon potential (OGEP)[4].

$$H = K + V_{CONF} + V_{OGEP} \qquad (1)$$

The kinetic energy part (K) is the sum of the kinetic energies including the rest mass minus the kinetic energy of the center of mass motion (CM) of the total system, i.e.,

$$K = \left[\sum_{i=1}^{2} M_i + \frac{P_i^2}{2M_i}\right] - K_{\rm cm}, \qquad (2)$$

Among different types of potential model , here we are going to introduce a new type of potential for the mesonic system that is the linear confining and the Hulthen potential $V_H$ is defined as the

$$V_H(r) = -Ze^2\mu \frac{\exp(\frac{-r}{\mu})}{1-\exp(\frac{-r}{\mu})} \qquad (3)$$

Where $\mu_0$ is a constant and $\mu$ is the screening parameter, determining the range for Hulthen potential. The Hulthen potential displays a typical property of the screening effect of a Coulomb-type interaction near the origin($r \to 0$), but it approaches to zero exponentially in the asymptotic region for $r \to \infty$. Hence in the limit $r \to 0$ the Hulthen potential behaves like coulomb -like potential with the strong coupling constant $\alpha_s$ is given by $V_H \simeq \frac{-4\alpha_s}{3r}$. Where $\alpha_s$ is the running coupling constant [2]. For our model we have chosen the linear confinement potential which represents the non perturbative effect of QCD that confines quarks within the color singlet system [4].

$$V_{CONF}(\vec{r}_{ij}) = -a_c r_{ij} \vec{\lambda}_i \cdot \vec{\lambda}_j \qquad (4)$$

where $a_c$ is the confinement strength and $\lambda_i$ and $\lambda_j$ are the generators of the color SU(3) group for the $i$th and $j$th quarks. The one gluon exchange potential is given by

$$V_{OGEP} = V_H(r) + V_{SD}(r) \qquad (5)$$

where the spin dependent potential $V_{SD}$ is introduced as an additional term to the potential to take into the account the spin-orbit and spin-spin interactions, causing the splitting of the $nL$ levels.

*Electronic address: praviphysics@gmail.com

$$V_{SD}(r) = \left(\frac{L\cdot S_c}{2m_c^2} + \frac{L\cdot S_b}{2m_b^2}\right)\left(-\frac{dV(r)}{rdr} + \frac{8}{3}\alpha_s\frac{1}{r^3}\right) + \frac{4}{3}\alpha_s\frac{1}{m_c m_b}\frac{L\cdot S}{r^3} + \frac{4}{3}\alpha_s\frac{2}{3m_c m_b}S_c\cdot S_b 4\pi\delta(r) + \frac{4}{3}\alpha_s\frac{1}{3m_c m_b}\left[3(S_c\cdot n)(S_b\cdot n) - S_c\cdot S_b\right]\frac{1}{r^3} \quad (6)$$

The central part of the two-body potential due to OGEP is [4],

$$V_{OGEP}(\vec{r}_{ij}) = \frac{\alpha_s}{4}\vec{\lambda}_i\cdot\vec{\lambda}_j \left[\frac{1}{r_{ij}} - \frac{\pi}{M_i M_j}(1+\frac{2}{3}\vec{\sigma}_i\cdot\vec{\sigma}_j)\delta(\vec{r}_{ij})\right] \quad (7)$$

In this model, the Hamiltonian of the spin-orbit term can be decomposed into symmetric and antisymmetric part. The antisymmetric part gives rise to the spin orbit mixing of heavy-light meson. For the case of quark and antiquark of unequal mass charge conjugation parity is no longer a good quantum number so that states with different total spins but with the same total angular momentum.

### Results and Conclusion

The variational technique represents a completely different way of getting approximate energies and wave functions for quantum mechanical systems. We construct a 5X5 Hamiltonian matrix for $B_s$ meson in the harmonic oscillator basis. In our calculation, the product of the quark-antiquark oscillator wave functions are expressed in terms of oscillator wave functions corresponding to the relative and center of mass coordinates. The masses of the $B_s$ mesons after diagonalization for successive values of $n_{max}$ Table I shows our current estimates compared to the experimental values and the non relativistic approach (Godfrey-Isgur Quark Model(GI)).

TABLE I: Masses of $B(b\bar{q}, q\in u,s)$ Spectrum in MeV

| $n\ ^{2S+1}L_J$ | b | Present Work | Exp | GI |
|---|---|---|---|---|
| $1\ ^1P$ | 0.925 | 5841 | | 5857 |
| $2\ ^1P$ | 0.980 | 6299 | | 6279 |
| $3\ ^1P$ | 1.012 | 6717 | | 6635 |
| $1\ ^3P_0$ | 0.912 | 5846 | $5710\pm 20$ | 5831 |
| $2\ ^3P_0$ | 0.985 | 6305 | | 6279 |
| $3\ ^3P_0$ | 1.045 | 6725 | | 6639 |
| $1\ ^3P'$ | 0.942 | 5848 | $5723.5\pm 2.0$ | 5861 |
| $2\ ^3P'$ | 0.986 | 6290 | | 6296 |
| $3\ ^3P'$ | 1.015 | 6669 | | 6650 |
| $1\ ^3P_2$ | 0.948 | 5855 | $5743\pm 5$ | 5876 |
| $2\ ^3P_2$ | 1.006 | 6322 | | 6295 |
| $3\ ^3P_2$ | 1.075 | 6785 | | 6648 |
| $1\ ^1D$ | 0.978 | 6159 | | 6169 |
| $2\ ^1D$ | 1.006 | 6533 | | 6526 |
| $3\ ^1D$ | 1.035 | 6841 | | 6841 |
| $1\ ^3D_1$ | 0.968 | 6124 | | 6182 |
| $2\ ^3D_1$ | 1.004 | 6517 | | 6542 |
| $3\ ^3D_1$ | 1.011 | 6827 | | 6855 |
| $1\ ^3D'$ | 0.998 | 6161 | | 6196 |
| $2\ ^3D'$ | 1.018 | 6535 | | 6553 |
| $3\ ^3D'$ | 1.015 | 6843 | | 6864 |
| $1\ ^3D_3$ | 1.008 | 6184 | | 6196 |
| $2\ ^3D_3$ | 1.0198 | 6545 | | 6535 |
| $3\ ^3D_3$ | 1.012 | 6829 | | 6849 |

[a]The nonrelativistic approach (Godfrey-Isgur Quark model)

### References


[1] Godfrey, Stephen and Moats, Kenneth and Swanson, Eric S.Phys. Rev. D.**94**, 054025, (2016)

[2] Bhaghyesh, K B Vijaya Kumar, Antony Prakash Monteiro, J. Phys.G **38**,085001, (2012)

[3] Hassanabadi, H. and Ghafourian,Few-Body Systems,**57**,254, (2016)

[4] Antony Prakash Monteiro and Manjunath Bhat and K. B. Vijaya Kumar International Journal of Modern Physics A,**32**, 1750021,(2017)