Mass Spectra of Orbitally Excited Charmonium in a Non Relativistic Quark model with an Instanton Potential

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Introduction

Charmonia are bound states of a charm and an anticharm quark ($c\bar{c}$), and represent an important testing ground for the properties of the strong interaction. There has been a great progress in the observation of the charmonium states from past few years. The discovery of the first charmonium state J/ψ has revolutionized the field of hadron spectroscopy. This lead to a clear understanding of the prevailing theory of particle physics. Several quarkonium states have been observed after the discovery of the charmonium state J/ψ at BNL and SLAC[1]. The first observation of singlet ground state of charmonium η_c was done by Mark II and crystal Ball experiments in the radiative decays of J/ψ and ψ'[1]. The discoveries of conventional states $h_c(1P)$, $h_c(2P)$, $\chi_c(1P)$, $\chi_c(2P)$, $\eta_c(1S)$ and the observation of the exotic states like X(3872), X(3915), Y(4260), Z(3930) at Belle, BaBar, LHC, BESIII,CLEO, etc have lead to renewed interest in quarkonium physics[1]. These new observations have given a deeper understanding of the charmonium physics and have unraveled many mysteries. Charmonium system is a powerful tool for the study of forces between quarks in QCD in non-perturbative regime. Studies of charmonia production can improve our understanding of heavy quark production and the formation of bound states. The exploration and understanding of the substructure of hadrons, presented in terms of quarks and gluons by quantum chromodynamics(QCD), has lead to a considerable progress in the study of charmonium states.

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Theoretical Background

In a potential approach the entire dynamics is governed by a Hamiltonian which is composed of a kinetic energy term K and a potential energy term V that takes into account the interaction between the quark and the antiquark,

$$H = K + V. \quad (1)$$

The mass spectra of the charmonium states and their decays are obtained, using the heavy-quark potential derived from the instanton vacuum $V_{Q\bar{Q}}(\vec{r})$ with the confining $(V_{conf}(\vec{r}))$ and Coulomb potentials $(V_{coul}(\vec{r}))$. The Kinetic energy is given by,[2]

$$K = M + \frac{p^2}{2\mu} \quad (2)$$

The potential energy V is given by,

$$V(\vec{r}) = V_{Q\bar{Q}}(\vec{r}) + V_{conf}(\vec{r}) + V_{coul}(\vec{r}) \quad (3)$$

The explicit form of the central potential from the instanton vacuum is[3]

$$V_C(\vec{r}) \simeq \frac{4\pi\bar{\rho}^3}{\bar{R}^4 N_c}\left(1.345\frac{r^2}{\bar{\rho}^2} - 0.501\frac{r^4}{\bar{\rho}^4}\right) \quad (4)$$

$$V_C(\vec{r}) \simeq 2\Delta M_Q - \frac{g_{np}}{r} \quad (5)$$

where the second term can be understood as a non perturbative contribution to the perturbative one gluon exchange potential at large r.

The non perturbative coupling constant g_{np} could be regarded as non perturbative correction to the strong coupling constant α_s. When r tends to infinity the potential is saturated at the value $2\Delta M_Q$ which implies that the instanton vacuum can not explain the quark confinement. ΔM_Q is the correction to the heavy-quark mass from the instanton vacuum [3]. The spin-dependent potentials are,

$$V_{SS}(\vec{r}) = \frac{1}{3{m_Q}^2}\nabla^2 V_C(\vec{r})$$

$$V_{LS}(\vec{r}) = \frac{1}{2{m_Q}^2}\frac{1}{r}\frac{dV_C(\vec{r})}{dr} \quad (6)$$

$$V_T(\vec{r}) = \frac{1}{3{m_Q}^2}\left(\frac{1}{r}\frac{dV_C(\vec{r})}{dr} - \frac{d^2V_C(\vec{r})}{dr^2}\right)$$

The expectation values of $< \vec{S}_Q \cdot \vec{S}_{\bar{Q}} >$ depends on the total spin $(\vec{S})$ of the meson, The confinement term represents the non perturbative effect of QCD which includes the spin-independent linear confinement term[4]

$$V_{conf}(\vec{r}) = -\left[\frac{3}{4}V_0 + \frac{3}{4}cr\right]F_1 \cdot F_2 \quad (7)$$

The coulomb-like (perturbative) one gluon exchange part of the potential is given by

$$V_{coul}(\vec{r}) = \frac{-4\alpha_s}{3r} \quad (8)$$

with the running strong coupling constant α_s,

Conclusions and Results

There are eight parameters in our model. These are the mass of charm quark m_c, the confinement strength c, the harmonic oscillator size parameter b, the coupling constant α_s. The constant parameters are ρ, $\bar{R}$, N_C and V_0.

$$m_c = 1475 \text{ MeV}; \quad b = 0.29 \text{ fm};$$

$$\alpha_s = 0.3; \quad c = 65 \text{ MeV fm}^{-1};$$

$$V_0 = -125 \text{ MeV};$$

We construct a 5X5 Hamiltonian matrix for low lying charmonium meson in the harmonic oscillator basis. In our calculation, the product of the quark-antiquark oscillator wave functions are expressed in terms of oscillator wave functions corresponding to the relative and centre of mass coordinates. The excited charmonium states are significantly influenced from systematic uncertainties, which are mainly due to the lack of knowledge of the long-range part of the spin-independent potential. Table I shows our current estimates compared to the experimental values and Martin like potential model.

TABLE I: Masses of nP and nD orbitally excited Charmonium States

$n^{2S+1}L_J$	Name	M_{exp}	Our Model	[5]
1^1P_1	$h_c(1P)$	3522	3525.38±0.11	3520
2^1P_1	$h_c(2P)$	3955	....	3960
1^3P_0	$\chi_{c0}(1P)$	3427	3414.75±0.31	3440
2^3P_0	$\chi_{c0}(2P)$	3605	3915±3	3920
1^3P_1	$\chi_{c1}(1P)$	3503	3510.66±0.07	3510
2^3P_1	$\chi_{c1}(2P)$	3902	3872	3950
1^3P_2	$\chi_{c2}(1P)$	3552	3553.20±0.09	3550
2^3P_2	$\chi_{c2}(2P)$	4025	3927.2±2.6	3980
1^1D_2	$\eta_{c2}(1D)$	3811	....	3840
2^1D_2	$\eta_{c2}(2D)$	4170	....	4210
1^3D_1	$\psi_1(1D)$	3788	3778	3820
2^3D_1	$\psi_1(2D)$	4182	4191±5	4190
1^3D_2	$\psi_2(1D)$	3834	3823	3840
2^3D_2	$\psi_2(2D)$	4276	....	4210
1^3D_3	$\psi_2(1D)$	3849	....	....
2^3D_3	$\psi_2(2D)$	4229	....	....

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