Annihilation Decays of Charmonium in a Non Relativistic Quark Model with an Instanton Potential

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Introduction

Charmonia are bound states of a charm and an anticharm quark ($c\bar{c}$), and represent an important testing ground for the properties of the strong interaction. We investigate the spectrum and decay rates of charmonium states within the framework of the non relativistic quark model by employing a Coulomb like potential from the perturbative one gluon exchange and the linear confining potential along with the potential derived form instanton vacuum [3] to account for the hyperfine mass splitting of charmonium states. We predict, two-photon, leptonic and two-gluon decay rates of low lying charmonium states. An overall agreement is obtained with the experimental masses and decay widths. We believe that a detailed experimental investigation of the spectrum of excited charmonia states and their decay properties will considerably improve our understanding of the non perturbative aspects of QCD.[2] Using the Van-Royen-Weisskopf relation we have calculated annihilation decay widths like leptonic decay width, two -photon and two gluon decay widths with the inclusion of radiative corrections.

Annihilation Decays

The annihilation decays of some charmonium states into gluons and light quarks make significant contributions to the total decay width of the states. The annihilation decays allow us to determine wave function at the origin. The annihilation decays of some $c\bar{c}$ states into photons can be used as a tool for the production and identification of resonances.[1]

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1. Two Gluon Decays

The charmonium states $^1S_0$, $^3P_0$, $^3P_2$ and $^1D_2$ can decay into two gluons, which account for a substantial portion of the hadronic decays for states below $c\bar{c}$ threshold. The two gluon decay widths are sensitive to the behaviour of the $q\bar{q}$ wave function and its derivatives near the origin. The two gluon decay widths are given by.

$$\Gamma(n\ ^1S_0 \to 2g) = \frac{2\alpha_s^2}{3m_c^2}|R_{nS}(0)|^2\left(1+\frac{4.8\alpha_s}{\pi}\right) \tag{1}$$

$$\Gamma(n\ ^3P_0 \to 2g) = \frac{6\alpha_s^2}{m_c^4}|R'_{nP}(0)|^2\left(1+\frac{9.5\alpha_s}{\pi}\right) \tag{2}$$

$$\Gamma(n\ ^3P_2 \to 2g) = \frac{8\alpha_s^2}{5m_c^4}|R'_{nP}(0)|^2\left(1-\frac{2.2\alpha_s}{\pi}\right) \tag{3}$$

$$\Gamma(n\ ^1D_2 \to 2g) = \frac{2\alpha_s^2}{3\pi m_c^6}|R''_{nD}(0)|^2 \tag{4}$$

2. Two Photon Decays

The $q\bar{q}$ quark pair in charge conjugation even states with $J \neq 1$ can annihilate into two photons. The expressions for the decay rates of $n\ ^1S_0$, $n\ ^3P_0$ and $n\ ^3P_2$ states into two photons with the first order QCD radiative corrections are given by[1]

$$\Gamma(n\ ^1S_0 \to \gamma\gamma) = \frac{3e_b^4\alpha^2}{m_c^2}|R_{nS}(0)|^2\left(1-\frac{3.4\alpha_s}{\pi}\right)$$

$$\Gamma(n\ ^3P_0 \to \gamma\gamma) = \frac{27e_b^4\alpha^2}{m_c^4}|R'_{nP}(0)|^2\left(1+\frac{0.2\alpha_s}{\pi}\right)$$

$$\Gamma(n\ ^3P_2 \to \gamma\gamma) = \frac{36e_b^4\alpha^2}{5m_c^4}|R'_{nP}(0)|^2\left(1-\frac{16\alpha_s}{\pi}\right)$$

The two photon decay widths of P wave charmonium states depend on the derivative of the radial wave function at the origin.

3. Leptonic Decays

The decay of vector meson into charged leptons proceeds through the virtual photon ($q\bar{q} \to l^+l^-$ where $l = e^-, \mu^-, \tau^-$). The $^3S_1$ and $^3D_1$ states have quantum numbers of a virtual photon, $J^{PC} = 1^{--}$ and can annihilate into lepton pairs through one photon. The leptonic decay width of the vector meson ($^3S_1$ charmonium) including first order radiative QCD correction is given by [1]

$$\Gamma(n\ ^3S_1 \to e^+e^-) = \frac{4\alpha^2 e_c^2 |R_{nS}(0)|^2}{M_{nS}^2} \left(1 - \frac{16\alpha_s}{3\pi}\right) \quad (5)$$

where $\alpha \approx \frac{1}{137}$ is the fine structure constant, M_{nS} is the mass of the decaying charmonium state and $e_c = 2/3$ is the charge of the charm quark in units of the electron charge. For D wave charmonium states the leptonic decay width with leading order QCD correction is given by

$$\Gamma(n\ ^3D_1 \to e^+e^-) = \frac{25\alpha^2 e_c^2 |R''_{nD}(0)|^2}{2m_c^4 M_{nD}^2} \left(1 - \frac{16\alpha_s}{3\pi}\right) \quad (6)$$

where M_{nD} is the mass of the decaying charmonium state.

Results and Conclusion

The model wave function, model parameters and the masses of the low lying charmonia states obtained have been used to study the decay properties of low lying charmonium. Using the predicted masses and the radial wave function at the origin, leptonic decays, two photon and two gluon decays are computed using the Van Royen-Weisskopf relation. The calculated values including the correction factor agree with the experimental values within a few MeV. These results also demonstrate the importance of the QCD correction factors including the decay constants and other short range effects using the potential model. The predicted decay widths are comparable with the experimental values for the low lying charmonia states. From our calculations, we conclude that the inclusion of QCD correction factors are of importance for obtaining accurate results for spectra and decay rates.

TABLE I: Leptonic Decay widths (keV)

State	Present Work $\Gamma_{l^+l^-}$	Exp $\Gamma_{l^+l^-}$	[4]
J/ψ	3.112	5.55±0.14	3.589
$\psi(2S)$	2.197	2.33 ± 0.07	1.440
$\psi(3S)$	1.701	0.86 ± 0.07	0.975
1^3D_1	0.275	0.242 ± 0.030	0.096
2^3D_1	0.223	0.83 ± 0.07	0.112

TABLE II: Two-Photon Decay widths (keV)

State	Present Work Γ	ExpΓ	[4]
$\eta_c(1S)$	6.96	7.2 ± 0.7	6.812
$\eta_c(2S)$	10.45	7.0 ± 3.5	2.625
$\eta_c(3S)$	1.03		1.760
$\chi_{c0}(1P)$	13.43	2.36 ± 0.35	2.119
$\chi_{c0}(2P)$	2.67		1.308
$\chi_{c2}(1P)$	1.72	0.66 ± 0.07	0.261
$\chi_{c2}(2P)$	0.343		0.168

TABLE III: Two-Gluon Decay widths (MeV)

State	Present Work Γ	Exp	[4]
$\eta_c(1S)$	28.60	28.6 ± 2.2	22.048
$\eta_c(2S)$	42.90	14 ± 7	8.496
$\eta_c(3S)$	4.26		5.696
$\chi_{c0}(1P)$	47.76	10.3 ± 0.6	6.114
$\chi_{c0}(2P)$	9.50		3.775
$\chi_{c2}(1P)$	5.27	1.97 ± 0.11	0.633
$\chi_{c2}(2P)$	1.04		0.401
1^1D_2	0.637		0.014
2^1D_2	0.177		0.012

References

[1] W. Kwong, P. B. Mackenzie, R. Rosenfeld, J. L. Rosner, Quarkonium annihilation rates, Phys. Rev. D **37** (1988) 32103215.

[2] K. Olive, et al., Review of particle physics, Chinese Physics C 38 **(9)** (2014) 090001.

[3] U. Yakhshiev, et al., Instanton effects on the heavy-quark static potential, arXiv preprint arXiv:1602.06074.

[4] Bhaghyesh, K. B. Vijyaya Kumar, Charmonium spectra and decays in a semirelativistic model, International Journal of Modern Physics A 27 **(22)** (2012)