Radiative Decays of Charmonium In a Non Relativistic Quark Model with an Instanton Potential

A P Monteiro[1],[*] Praveen P D'Souza [1] [2], and K. B. Vijaya Kumar [2]
[1] *P.G Department of Physics, St Philomena College, Darbe, Puttur-574202,INDIA and*
[2] *Department of Physics, Mangalore University,*
Mangalagangothri P.O., Mangalore - 574199, INDIA

Introduction

Charmonia are bound states of a charm and an anticharm quark ($c\bar{c}$), and represent an important testing ground for the properties of the strong interaction. We investigate the spectrum and decay rates of charmonium states within the framework of the non relativistic quark model by employing a Coulomb like potential from the perturbative one gluon exchange and the linear confining potential along with the potential derived form instanton vacuum [3] to account for the hyperfine mass splitting of charmonium states. We predict radiative E1, M1, decay rates. Charmonium system is a powerful tool for the study of forces between quarks in QCD in non-perturbative regime. The main achievement of the introduction of instantons in hadron spectroscopy is the resolution of the $U_A(1)$ problem, which leads for a good prediction of the masses of η and η' mesons. The recent results from B factories and new programs at BES, CLEO, and GSI have led to a resurgence of interest in the physics of charmonia. We believe that a detailed experimental investigation of the spectrum of excited charmonia states and their decay properties will considerably improve our understanding of the non-perturbative aspects of QCD.[2]

Radiative Transitions

In calculating the radiative decay widths, we have assumed that in the non relativistic limit, the dipole radial matrix elements are independent of J, i.e all states within the same angular momentum multiplet have the same wave function Radiative transitions could play an important role in the discovery and identification of charmonium states[1]. They are sensitive to the internal structure of states, in particular to $^3L_L - ^1L_L$ mixing for states with J = L.

1. *E1 Transitions*

The partial width for an E_1 radiative transition between states in the non relativistic quark model is given by,

$$\Gamma(i \to f + \gamma) = \frac{4\alpha e_c^2}{3}(2J_f+1)S^E_{if}k_0^3\,|\mathcal{E}_{if}|^2 \tag{1}$$

where k_0 is the energy of the emitted photon $k_0 = m_i - m_f$,α is the fine structure constant. $e_c = 2/3$ is the charge of the c quark in units of $|e|$. m_i and m_f are the masses of initial and final mesons. The statistical factor $S^E_{if} = \max(\mathrm{L_i},\mathrm{L_f})\left\{\begin{matrix} J_i & 1 & J_f \\ L_f & S & L_i \end{matrix}\right\}^2$, J_i, J_f are the total angular momentum of initial and final mesons, L_i, L_f are the orbital angular momentum of initial and final mesons and S is the spin of initial meson.

$$\begin{aligned}\mathcal{E}_{if} = \frac{3}{k_0}\int_0^\infty & r^3 R_{nl}(r)R'_{nl}(r)dr \\ & \left[\frac{k_0 r}{2} j_0\left(\frac{k_0 r}{2}\right) - j_1\left(\frac{k_0 r}{2}\right)\right]\end{aligned} \tag{2}$$

is the radial overlap integral which has the dimension of length, $R_{nl}(r)$ and $R'_{nl}(r)$ are the normalized radial wave functions for the corresponding states and j_0 and j_1 are spherical Bessel functions.

2. *M1 Transitions*

Radiative transitions which flip spin are described by magnetic dipole (M1) transitions.

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[*] Electronic address: `praviphysics@gmail.com`

The rates for magnetic dipole transitions between S-wave $c\bar{c}$ states are given in the non relativistic approximation by [4]

$$\Gamma(i \to f + \gamma) = \frac{4\alpha e_c^2}{3m_c^2} \frac{2J_f + 1}{2L_i + 1} \delta_{L_i L_f} \delta_{S_i S_f} k_0^3 |M_{if}(r)|^2 \quad (3)$$

where M_{if} is the radial overlap integral which has the dimension of length,

$$M_{if} = \int_0^\infty 4\pi r^3 R_{nl}(r) j_0(kr/2) R'_{nl}(r) dr \quad (4)$$

is the overlap integral for unit operator between the coordinate wave functions of the initial and the final meson states, $j_0(kr/2)$ is the spherical Bessel function, m_c is the mass of charm quark. J_f is the total angular momentum of final meson state, L_i is the orbital angular momentum of the initial meson state.

Results and Conclusion

Radiative decays of excited charmonium states are powerful tool to study the internal structure of the mesons. The model wave function, model parameters and the masses of the charmonia states obtained have been used to study the decay properties of charmonium. As a consequence of the higher excited states of charmonium systems being known, it is natural to study their E1 and M1 transitions rates. Since very few of the pseudoscalar excited states (n^1S_1) are known, the experimental data for M1 transitions are less understood compared to E1 transitions.However, there are some differences in the predictions due to differences in phase space arising from different mass predictions and also from the wave function effects. We find our results are compatible with other theoretical model values for most of the channels. The recent results from B factories and new programs at BES, CLEO, and GSI have led to a resurgence of interest in the physics of charmonia. We believe that a detailed experimental investigation of the spectrum of excited charmonia states and their decay properties will considerably improve our understanding of the nonperturbative aspects of QCD.

TABLE I: E1 Transition rates in MeV

E1 Transition	k	This Work Γ	Γ_{Expt}	[?]
$1^3P_0 \to 1^3S_1$	383	146.9	119.5±8	152
$1^3P_1 \to 1^3S_1$	409	264.825	295.8±13	314
$1^3P_1 \to 1^1S_0$	520	517.322		498
$1^3P_2 \to 1^3S_1$	458	364.186	384.2±16	424
$2^3S_1 \to 1^3P_0$	245	12.94		63
$2^1S_0 \to 1^1P_1$	108	10.46		49
$2^3S_1 \to 1^3P_1$	169	13.144	28.0±1.2	54
$2^3S_1 \to 1^3P_2$	120	7.95	26.6±1.1	38
$1^3D_1 \to 1^3P_0$	361	200.8	199.3±25	403
$1^3D_1 \to 1^3P_1$	285	76.65	79.2±16	125
$1^3D_1 \to 1^3P_2$	236	2.95	3.88	< 24.6
$1^3D_2 \to 1^3P_1$	331	211.976		307
$1^3D_2 \to 1^3P_2$	282	44.605		64
$1^1D_2 \to 1^1P_1$	289	191.51		339
$1^3D_3 \to 1^3P_2$	297	207.195		272

TABLE II: M1 Transition rates MeV

(M_1)Transition	k	This Work Γ	Γ_{Expt}	[?]
$1^3S_1 \to 1^1S_0$	113	3.17	1.58±0.37	2.9
$2^3S_1 \to 2^1S_0$	43	0.175	0.14±0.02	0.21
$2^3S_1 \to 1^1S_0$	705	2.02	0.97±0.02	4.6
$2^3S_1 \to 1^3S_1$	589	0.600		7.9
$3^3S_1 \to 2^1S_0$	546	1.117		0.61
$3^1S_0 \to 2^3S_1$	514	0.744		1.3
$3^1S_0 \to 1^3S_1$	1103	0.620		6.3
$2^1P_1 \to 1^3P_2$	624	29.73		0.07
$2^1P_1 \to 1^3P_1$	574	10.172		0.05
$2^1P_1 \to 1^3P_0$	653	8.05		0.03
$2^3P_2 \to 1^1P_1$	162	0.0016		0.6
$2^3P_1 \to 1^1P_1$	475	2.811		0.05
$2^3P_0 \to 1^1P_1$	186	0.0042		0.02

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