Coherence length & Fermi Momentum in heavy and light meson condensation

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Introduction

The very well known NJL model [1], suggested by Nambu, Jona, Lasinio is a dynamic model of elementary particle based on an analogy with superconductivity. The quasiparticle excitation which appears in the BCS Theory of superconductivity has a close analogy with the properties of Dirac paticle which leads to a NJL model description of the elementary particles. The vacuum can be described as the sea of virtual qq pairs similar to the Cooper pairs in a superconductor and such vacuum represents a background of a coherent system of particles and a qq condensate is formed. The NJL model, which was originally a model of interacting nucleons, has been reinterpreted as a schematic quark model. It is widely used to study the the properties of meson and diquark condensates.

In this present article we have presented the expression for the gap energy of <qq-bar> condensate in the framework of the NJL model for meson condensates. Also the variation of the coherence length with the Fermi momentum for light and heavy meson condensate have been studied.

NJL model and related mathematical expressions

The effective Lagrangian describing the local quark-antiquark interaction in the two flavor NJL model is described [2] as

$$L = L_0 + L_{int} \tag{1}$$

$$L_0 = \bar{q}(i\gamma - m_0)q \tag{2}$$

$$L_{\text{int}} = G[(q\bar{q})^2 + (\bar{q}i\gamma_5 q)^2] \tag{3}$$

Where G is the dimensional strong coupling constant with dimension GeV^{-2}.
The <qq-bar> condensation in the momentum space can be written like:

$$<\bar{q}q> = -12i\int \frac{d^4 p(\Sigma + m_0)}{(2\pi)^4 (p^2 - (\Sigma + m_0)^2 + i\in)} \tag{4}$$

The integral becomes finite with the cut-off Λ. With, $\sum +m_0 = m$, the constituent quark mass, we arrive at the gap equation

$$m = m_0 - 2G\bar{q}q \tag{5}$$

Assuming the cut-off parameter Λ as the Fermi momentum 'p' of the corresponding condensate, we arrive at the expression of the gap energy as

$$\Delta = m - m_0 = \frac{3Gmp_f^2}{\pi^2}\left(\sqrt{1+\frac{m^2}{p_f^2}} - \frac{m^2}{p_f^2}\ln\frac{p_f^2 + \sqrt{p_f^2+m^2}}{m}\right) \tag{6}$$

Δ must be a positive quantity for the meaningful interpretation of gap energy. With $x = p_f/m < 1$ and using

$\ln[x + (x^2+1)^{1/2}] = x - x^3/6 + 3x^5/40 - \ldots\ldots$, the expression (6) may be expanded to obtain Δ as

$$\Delta = \frac{3Gm}{\pi^2}\left(\frac{2}{3}m^2x^3 - \frac{1}{5}m^2x^5 + \ldots\ldots\ldots\ldots\right) \tag{7}$$

Thus we have come across an expression relating the Fermi momentum and gap energy.

Results & Discussions

It is well known that at low energy the hadronic phase is realized with dynamically broken chiral symmetry by condensation of quark-antiquark pairs. In the present work, the superconducting order parameter or excitation gap has been estimated with the Fermi momentum as cut-off.

With $g^2=G\Lambda'^2$ [2] where "g" is the dimensionless coupling constant. We have considered the strong coupling limit $g^2/4p\geq1$ [2] and used "g" ~=$\sqrt{(4\pi)}$=3.54. We have plotted the Fermi momentum p_f correlation length ξ ($1/\Delta$) graph for light and heavy condensates, and have shown it in figure with Λ'=1.25 GeV for the light sector [3] and 1GeV for the heavy sector [4].

We have observed that the p=0.272 GeV yields the coherence length 0.25 fm for <ss-bar> condensate whereas p =0.254 GeV yields the same value of coherence length (0.25 fm) for heavy condensates [5].

In the present work it also has been observed that for the coherence length to be ~ 1fm, the Fermi momentum should be 0.176 GeV and 0.172 GeV for <uu-bar> and <ss-bar> respectively, whereas, for the heavy meson condensate, p_f=0.142 GeV yields the coherence length ~ 1 fm [5].

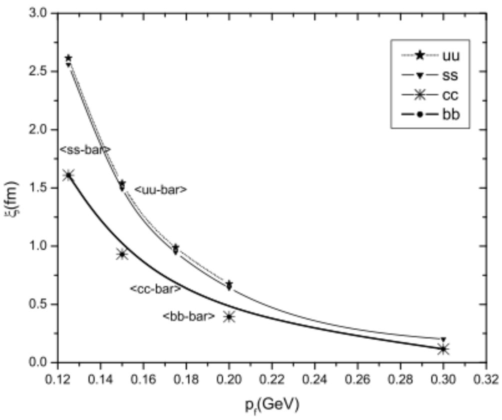

Fig. 1. The coherence length (ξ) of mesons as a function of the Fermi momentum.

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In the present work we have studied the quark-antiquark condensates formation from vacuum and extracted the values of Fermi momentum studying the coherence length for both light- and heavy-meson condensates. It may be mentioned that the Fermi momentum is a very important quantity, particularly for the study of CKM matrix elements. We shall make a study on diquarks in the NJL model in the context of the quasiparticle picture of diquarks in our future works.

References

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