Azimuthal Anisotropy in Heavy-Ion Collisions using Non-extensive Statistics in Boltzmann Transport Equation

S. Tripathy,∗ S. K. Tiwari, M. Younus, and R. Sahoo
Discipline of Physics, School of Basic Sciences, 
Indian Institute of Technology Indore, Indore- 453552, INDIA

Introduction

One of the major goals in heavy-ion physics is to understand the properties of Quark Gluon Plasma (QGP), a deconfined hot and dense state of quarks and gluons existed shortly after the Big Bang. In the present scenario, the high-energy particle accelerators are able to reach energies where this extremely dense nuclear matter can be probed for a short time. One of the most interesting observables to probe the QGP is the azimuthal anisotropy of the system, and this shows the measurement of the asymmetry of particle density in momentum space relative to the reaction plane. In non-central heavy-ion collisions, the overlap region resembles to an almond shape with the major axis assumed to be perpendicular to the reaction plane. Due to large fluctuations, the participant plane doesn’t necessarily remain the same as the reaction plane. When the system evolves, the anisotropy in the coordinate space is reflected in the momentum-space due to the pressure gradient. Thus, the particle distribution can be written as,

\[ E \frac{d^3N}{dp^3} = \frac{d^2N}{2\pi T dp dy} \left( 1 + \sum_{n=1}^{\infty} 2v_n \cos(n\phi) \right), \]

where, \( \phi \) is the azimuthal angle of a particle. The second harmonic coefficient is called as elliptic flow (\( v_2 \)) of the system, which signifies the transformation from geometric asymmetry to momentum-space asymmetry due to

the strong interactions of quarks and gluons.

Here, we follow our earlier works [1, 2] which use non-extensive statistics in Boltzmann Transport Equation (BTE). We represent the initial distribution of particles with the help of Tsallis power law distribution parameterized by the nonextensive parameter \( q \) and the Tsallis temperature \( T \), remembering the fact that their origin is due to hard scatterings. We use the initial distribution \( (f_{in}) \) with Relaxation Time Approximation (RTA) of the BTE and calculate the final distribution \( (f_{fin}) \). Then we calculate \( v_2 \) of the system using the final distribution in the definition of \( v_2 \).

Theoretical Formulation

The evolution of the particle distribution owing to its interaction with the medium particles can be studied through Boltzmann transport equation,

\[ \frac{df(x,p,t)}{dt} = \frac{\partial f}{\partial t} + \vec{v} \cdot \nabla_x f + \vec{F} \cdot \nabla_p f = C[f], \]

where \( f(x,p,t) \) is the distribution of particles which depends on position, momentum and time. \( \vec{v} \) is the velocity and \( \vec{F} \) is the external force. \( \nabla_x \) and \( \nabla_p \) are the partial derivatives with respect to position and momentum, respectively. \( C[f] \) is the collision term which encodes the interaction of the probe particles with the medium. We assume homogeneity of the system and absence of external force in BTE.

In RTA, the collision term can be expressed as,

\[ C[f] = -\frac{f - f_{eq}}{\tau} \]

Available online at www.sympnp.org/proceedings
where $f_{eq}$ is the Boltzmann-Gibbs Blast Wave (BGBW) function \[2\], $\tau$ is the relaxation time for the probe distribution function $f$. Solving the Eq. 2 in view of initial conditions i.e. at $t = 0, f = f_{in}$ and at $t = t_F, f = f_{fin}$; leads to,

\[ f_{fin} = f_{eq} + (f_{in} - f_{eq})e^{-\frac{t_f}{\tau}}, \tag{4} \]

where $t_f$ is the freeze-out time. Now, $v_2$ can be written as,

\[ v_2 = \frac{\int_0^{2\pi} f_{fin} \cos 2\phi \ d\phi}{\int_0^{2\pi} f_{fin} \ d\phi}, \tag{5} \]

where $\phi$ is the azimuthal angle in momentum space. Eq. 5 is used to describe the $v_2$ spectra for different particles in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV.

**Results and Discussion**

It is found that using BGBW function in the definition of $v_2$, we can not explain $v_2$ above transverse momentum ($p_T > 2$ GeV/c) as evident from Fig. 1. So, we use Eq. 5 to study the $v_2$ for various particles. Keeping the parameters free ($T, q, t_f/\tau$ etc.), we fit the $v_2$ spectra for $K_s^0$, $\phi$ and $\Lambda + \bar{\Lambda}$ [3] in peripheral (50-60)% Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV, which are shown in Fig. 2. The fitting gives a proper explanation of elliptic flow trend for different particles at low as well as intermediate $p_T$ ranges. The proposed model is able to explain the contrasting behaviour of baryons from mesons. It is also observed that the above formalism explains the $v_2$ better than the simple BGBW formalism, particularly after $p_T > 2$ GeV/c. A detailed study of parameters and fitting to other light flavoured particles will be shown in Ref. [4].

In summary, we have used a simple model where the RTA of BTE transports a power law inspired Tsallis distribution to an equilibrium BGBW. The model explains experimental data reasonably well, which indicates a major role of BTE with non-extensive statistics in heavy-ion collisions.

**Acknowledgements**

ST acknowledges the financial support by DST INSPIRE program of Govt. of India.

**References**


