Shear Viscosity and Isothermal Compressibility of a Hadron Gas in Non-extensive statistics

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Introduction

The space-time evolution of a system from non-equilibrium to equilibrium state is governed by its transport properties. In nuclear collisions, the spatial anisotropy gets converted to the momentum anisotropy of the produced particles [1]. The equilibration of momentum anisotropy is governed by the shear viscosity. The AdS/CFT calculations show that there is a lower bound to the value of shear viscosity to entropy density ratio (η/s). The lower bound (known as the Kovtun-Son-Starinets (KSS) bound) has been set to the value $1/4\pi$ in natural units [1]. The elliptic flow measurement at RHIC has found that the η/s is close to the KSS bound, which developed intense interest in transport properties of strongly interacting matter governed by Quantum Chromodynamics (QCD). Another interesting thermodynamic property is the isothermal compressibility (κ_T), which is used to define the equation of state of the system and quantify the softest point of the phase transition along with the location of the critical point.

Due to high multiplicities produced in high-energy collisions, the statistical models are more suitable to describe the particle production mechanism. Such a statistical description of transverse momentum (p_T) of final state particles produced in high-energy collisions has been proposed to follow a thermalized Boltzmann-Gibbs (BG) distribution. But, a finite degree of deviation from the equilibrium statistical description of p_T spectra has been observed at RHIC and LHC. Fortunately, for anomalous systems, where the usual ergodicity is violated such as the metastable states in large systems involving long range forces between particles, a generalized BG entropy has been introduced by C. Tsallis [1]. The fits to the particle spectra suggest deviations from $q = 1$, which means that the local equilibrium differs from the standard BG equilibrium and this equilibrium is termed as q-equilibrium. In Boltzmann Transport Equation (BTE) with BG distribution function, it is assumed that a non-equilibrium system relaxes to an equilibrium system after a certain relaxation time, τ. Here, we investigate a non-equilibrium system which dissipates energy and produces entropy and it does not relax to a BG equilibrium but to a local q-equilibrium.

We study the transport coefficients such as η/s and κ_T using the relativistic non-extensive Boltzmann transport equation (NBTE), where we employ the Relaxation Time Approximation (RTA) for the collision integral.

Theoretical Formulation

For a multi-component hadron gas the shear viscosity can be calculated as [2]:

$$\eta = \frac{1}{15T}\sum_a \int \frac{d^3p}{(2\pi)^3}\frac{p^4}{E_a^2}\left(\tau_a f_a^0 + \bar{\tau_a}\bar{f_a^0}\right), \quad (1)$$

where τ_a and f_a^0 are the relaxation time and distribution function for a^{th} hadron, respectively while bar stands for anti-hadron. E_a is the energy of a^{th} hadron. The entropy density of hadrons in Tsallis statistics is calculated by using the basic thermodynamical relation.

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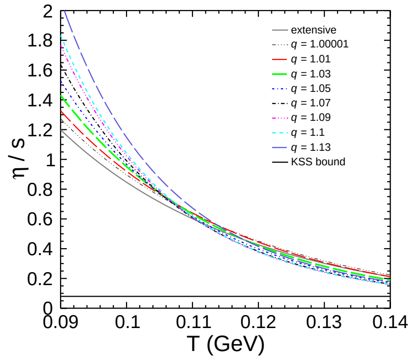

FIG. 1: Variations of shear viscosity to entropy density ratio (η/s) with respect to temperature (T) at various q-values.

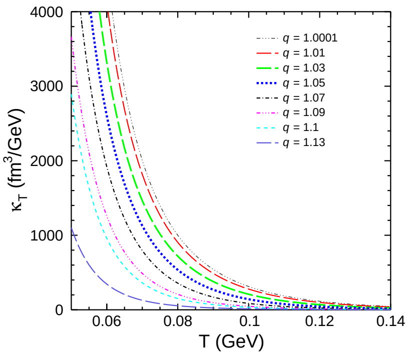

FIG. 2: The isothermal compressibility (κ_T) versus temperature at different q-values.

The isothermal compressibility (κ_T) is defined as,

$$\kappa_T = -\frac{1}{V}\frac{\partial V}{\partial P}, \tag{2}$$

where, V is the volume of the system. Again in terms of fluctuation and average number, isothermal compressibility can be expressed as [3, 4],

$$\left\langle (N - \langle N \rangle)^2 \right\rangle = var(N) = \frac{T \langle N \rangle^2}{V} \kappa_T. \tag{3}$$

Using the relation $\left\langle (N - \langle N \rangle)^2 \right\rangle = VT\frac{\partial n}{\partial \mu}$, Eq. 3 can further be reduced as,

$$\frac{1}{\kappa_T} = \sum_a \frac{n_{aq}^2}{\left(\frac{\partial n_{aq}}{\partial \mu}\right)}. \tag{4}$$

where, $\frac{\partial n_q}{\partial \mu}$ is given as [3],

$$\frac{\partial n_q}{\partial \mu} = \frac{gq}{T}\int \frac{d^3p}{(2\pi)^3}\left[1 + (q-1)\frac{E-\mu}{T}\right]^{\frac{1-2q}{q-1}}. \tag{5}$$

Results and Discussion

In Figure 1, we present the variations of shear viscosity to entropy density (η/s) ratio with respect to temperature (T) for hadrons calculated in non-extensive statistics at zero chemical potential (μ). We notice that η/s initially decreases rapidly with T for all the q-values while it becomes independent of q at higher T. Figure 2 shows the isothermal compressibility (κ_T) as a function of T for $\mu = 0$. We use Eq. 4 to calculate κ_T in non-extensive statistics for various q values. We observe that κ_T initially decreases rapidly with T and then saturates at higher temperatures. We find that κ_T saturates at different T for different q. This suggests that the criticality of system depends strongly on the q-values.

In summary, η/s and κ_T are sensitive to the non-extensivity of the system measured in terms of the q-parameter. Thus, it is very important to study the q-dependence of these observables towards locating the critical point.

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References

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