Properties of quarkonia in a strong magnetic field

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Introduction

Recent analysis suggests that a strong magnetic field is expected to be produced at very early stage of ultrarelativistic heavy ion collisions (URHIC), when the event is off-central. Heavy quark pairs (QQ) are also produced at the nascent stages of URHICs when the magnetic field is expected to be very strong. For example, charm-anti charm pairs are produced at a typical time of $t_{cc}(\approx 1/2m_c) \approx 0.1$ fm whereas the magnetic field is expected to be very strong typically up to $t_B \approx 0.2$ fm. Thus the coincidence of production time scales motivates us to investigate the properties of heavy quarkonia in the presence of strong magnetic field. For this purpose we have first calculated the gluon self-energy for a thermal QCD up to one-loop in a strong magnetic field in real-time formalism to calculate the effective gluon propagator, which in the static limit gives the dielectric permittivity, $\varepsilon(k)$ embodying the effects of strong magnetic field on thermal (QCD) medium. Finally the inverse Fourier transform of permittivity yields the desired heavy quark-antiquark potential, $V(r; T, B)$ and the binding energies for different quarkonium states are obtained by solving the Schrödinger equation with the above potential.

Screening mass in strong magnetic field

Since the medium modification to the potential in vacuum enters through the screening mass, which is obtained from the static limit of the longitudinal component of the gluon self-energy so using Keldysh representation we will first thermalize the vacuum quark propagator in the presence of magnetic field in real-time formalism, we have recently calculated the real part of gluon self energy up to one-loop [1],

$$\Pi_{00}(0, k_\perp, k_z) = \frac{g^2}{4\pi^2} \sum_f |qf B|e^{-\frac{k_z^2}{2\beta^2}} \int dp_z \left[ \frac{p_z n_p}{\omega_f k_z} + \frac{(p_z - k_z)n_q}{\omega_q k_z} - \frac{2m_f^2 n_p}{\omega_p k_z(2p_z - k_z)} + \frac{2m_q^2 n_q}{\omega_q k_z(2p_z - k_z)} \right].$$ (1)

Hence the Debye screening mass is thus obtained from the static limit of (1) as [2, 3]

$$m_D^2 = \frac{g^2}{4\pi^2 T} \sum_f |qf B| \int_0^\infty dk_z \frac{e^{\beta \sqrt{k_z^2 + m_f^2}}}{(1 + e^{\beta \sqrt{k_z^2 + m_f^2}})^2}.$$ (2)

Thus the presence of strong magnetic field makes the screening mass to depend on both magnetic field and temperature. However, it mainly depends on magnetic field due to the strong magnetic field limit ($eB >> T^2$), in fact, it becomes temperature-independent beyond a certain temperature. So the collective behavior of the medium is significantly affected by strong magnetic field, which in turn will affect the potential in figure 1.

Heavy quark potential in strong magnetic field

Since the transverse component of gluon self-energy vanishes in the static limit so the effective gluon propagator is obtained by the longitudinal component of the gluon self-energy (1) only in terms of the Debye mass

$$D_{11}^f(0, k) = \frac{1}{k^2 + m_D^2}.$$ (3)

As a result, the long-range of electrostatic color interaction gets screened. Thus the dielectric permittivity

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is obtained as
\[
e^{-1}(k) = \lim_{k_0 \to 0} k^2 D_1^2(k_0, k),
\]
(4)

Therefore the potential in vacuum (T=0) is corrected in the medium by the dielectric permittivity through its Fourier component, \( V(k) \) as
\[
V(r, T) = \int \frac{d^3k}{(2\pi)^{3/2}} \left( \frac{e^{i\mathbf{k} \cdot \mathbf{r}} - 1}{\epsilon(k)} \right) V(k),
\]
(5)

where an \( r \)-independent term is subtracted to renormalize the heavy quark free energy, which is the perturbative free energy of quarkonium at infinite separation.

Therefore the \( Q\bar{Q} \) potential in thermal QCD medium in presence of strong magnetic field is given by (with \( \hat{r} = r m_D \))
\[
V(r; T, B) = -\alpha m_D \left[ \frac{e^{-\hat{r}}}{\hat{r}} + 1 \right] + \frac{2\sigma}{m_D} \left[ \frac{(e^{-\hat{r}} - 1)}{\hat{r}} + 1 \right],
\]
(6)

where the first and second term represent the Coulombic and string term, respectively. Thus the potential (6) now depends on both temperature and magnetic field through the Debye mass. The nonlocal terms in the potential insure \( V(r, T) \) to reduce to the Cornell potential in \( T \to 0 \) limit, which are however needed in computing the masses of the quarkonium states and to compare with the free energy in lattice studies.

To see the effect of magnetic field on the potential between \( Q \) and \( \bar{Q} \), we have plotted the potential as a function of interquark distance in a pure thermal medium (absence of magnetic field) and thermal medium in presence of magnetic field in figure 1, after excluding the constant terms. We have found that the presence of strong magnetic field affects the linear string term more than the Coulomb term, as a result the total potential becomes more attractive, i.e. less screened as compared to the potential in pure thermal medium (denoted by solid line, \( B = 0 \)).

To investigate the properties of quarkonia in a strong magnetic field, the Schrödinger equation is solved by employing the temperature and magnetic field dependent potential (6) and obtain the binding energies for different quarkonium states by the energy eigenvalues. We found that the binding energies of quarkonia in a thermal medium are decreased due to the increasing ambient magnetic fields, thereby estimating the magnetic field beyond which the potential will be too weak to bind \( Q\bar{Q} \) together. The dissociation magnetic field for \( J/\psi \) is found to be approximately \( 10 m_\pi^2 \) whereas the excited state, \( \psi' \) is dissociated at much smaller magnetic field \( \sim m_\pi^2 \).

References