Solution of Balitsky-Kovchegov equation in linearized form

Mariyah Siddiqah*
Department of Physics, Aligarh Muslim University, Aligarh - 202002, UP, India.

Introduction

Quarks and guons interact via strong interactions and the theory of strong interactions is described by Quantum Chromodynamics(QCD). Experimentally it has been seen that the gluon density inside the proton(and simlarly inside the nucleus) grows rapidly when the Bjorken x (fraction of protons momentum carried by the gluon) decreases.This is equivalent to probing the hadron at high energy, and allows us to concider proton or nucleus as a medium of dense gluon matter known as Color Glass Condensate(CGC). At high energies any scattering event in the high energy collider experiment involves a rapidly growing cascade of gluons because of the reason that emitted gluons themselves emit further gluons occupying all the final-state phase space available to them. Number desity of partons as well as the cross section associated with them grows as the positive powers of energy. At higher energies partons not only split into further partons, but also recombine. overlaping of partons with each other leads to the areas of very high density. Recombination helps to form the thermodynamical balance with the multiple gluon production from the single gluon and also reduces the number of partons in the wave function, which ultimately leads to the origin of gluon saturation with the charracteristic momentum scale Q_s[1].Unitary of the S-matrix is restored by this gluon recombination which would have been voilated by an expnential growth of gluon multiplicities. The energy dependence of the observables in the high energy regime can be calculated through evolution equations which are derived from the QCD in the high energy limit.One of these evolution equations is Balitsky- Kovchegov(BK) equation.This equation was derived independently by Balitsky and Kovchegov. This equation has the advantage of being a closed equation for scattering amplitude.BK equation is considered as the simple and most accurate for describing the saturation region of QCD.

*Electronic address: shah.siddiqah@gmail.com

Muller Model

Energy dependence into the Dipole-nucleus amplitude comes through the Quantum evolution corrections. So for this we have to use the approch developed by the Muller, formulated in the transverse coordinate. it is more convenient to include satutation effects in this model as transverse coordinates do not change during the rapdity evolution. Emission of small-x gluon taken in the large-N_c limit splits the original dipole into two.The probability of emission of the soft gluon off this dipole is given by the formulla,

$$I_{\rm dip} \equiv \int d^2x_2 \; \frac{x_{10}^2}{x_{20}^2 x_{21}^2} \equiv \int \frac{(x-y)^2}{(x-z)^2(z-y)^2} d^2z \; , \quad (1)$$

The formalism for calculating the integration of this particular integral has been described in detail in our recent paper [2] . Where we have considered all the higher order terms of the integral that have been ignored earlier for some reasons.The Balitsky-Kovchegov equation in the large-N_c limit is defined by the following equation

$$\frac{\partial}{\partial Y} S(x_{01}, Y) = \frac{\alpha_s N_c}{2\pi^2} \int d^2x_2 \; \frac{x_{01}^2}{x_{02}^2 x_{21}^2} \times \; [S(x_{02}, Y) S(x_{12}, Y) - S(x_{01}, Y)] \, . \quad (2)$$

when $x_{20}, x_{21} > x_{10}$ we could also expect $S(x_{20})S(x_{21}) < S(x_{10})$, BK equation, an integro-differential equation in general, becomes first order partial differential equation of $S(x_{10}, Y)$, the solution of which is given as

$$S = S_0 \exp\left(\frac{1+2i\nu_0}{2\chi(0,\nu_0)}\left[\mathrm{Li}_2\left(-\lambda_1 x_{10}^2 Q_s^2(Y)\right)\right]\right) \quad , \qquad (3)$$

where Li_2 is dilogarithm function and $\lambda_1 (\approx 7.22)$ is a parameter which is to be fixed by the definition of Q_s.Thus the imaginary part of the dipole-nucleus amplitude for deep inelastic scattering of the dipole with a large nucleus is given by,

$$N(x_{10}, Y) = 1 - S_0 \exp\left(\frac{1+2i\nu_0}{2\chi(0,\nu_0)}\left[\mathrm{Li}_2\left(-\lambda_1 x_{10}^2 Q_s^2(Y)\right)\right]\right) \quad , \qquad (4)$$

In the limit $x_{10}Q_s << 1$, the above equation is in accordance with the McLerran-Venugopalan model which is gaussian in $\tau = x_{10Q_s(Y)}$

$$N_{\mathrm{MV}}(x_{10}, Y) = \quad 1 - S_{MV}(x_{10}, Y) = 1 - \exp\left(-\kappa \; x_{10}^2 Q_s^2(Y)\right) \qquad (5)$$

And in the limit $x_{10}Q_s >> 1$, the equation reproduces the Levin-Tuchin solution which is gaussian in a logrithm of $\tau = x_{10Q_s(Y)}$.

$$N_{\mathrm{LT}}(x_{10}, Y) = 1 - \exp\left(-\frac{1+2i\nu_0}{4\chi(0,\nu_0)} \ln^2\left[x_{10}^2 Q_s^2(Y)\right]\right), \qquad (6)$$

Equation(4) connects both the limit s($x_{10}Q_s << 1$ and $x_{10}Q_s >> 1$) smoothely, with a better accuracy.

Conclusion

In FIG.1 the dipole ampliude $N(x_{10}, Y)$ is ploted as a function of scaling variable $\tau = x_{10}Q_s(Y)$. Our new solution, Eq.(4), is compared with the numerical solutions of leading order (LO) Balitsky-Kovchegove equation for two sets of rescaled rapidity, $\alpha_s Y = 1.2, 2.4$ and one set of fixed coupling $\alpha_s = 0.2$. The solution is in better agreement with the numerical solutions of the full LO BK equation in a wide kinematic domain inside the saturation region.

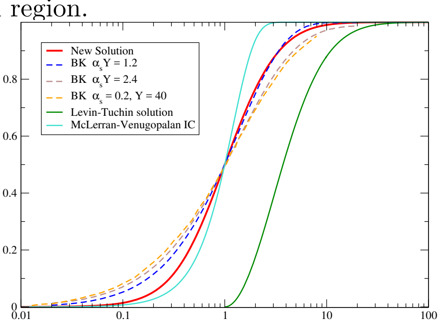

FIG. 1: The dipole amplitude $N(x_{10}, Y)$ as function of scaling variable $\tau = x_{10}Q_s(Y)$; new solution Eq.(4) compared with numerical solutions of leading order Balitsky-Kovchegove equation and the McLerran-Venugopalan initial condition Eq.(5) and Levin-Tuchin solution Eq.(6).

References

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