Simple textures for neutrino mass matrix with magic symmetry

Kanwaljeet S. Channey* and Sanjeev Kumar†
Department of Physics and Astrophysics, University of Delhi, Delhi - 110007, INDIA

Introduction

In the standard model (SM), neutrino flavor states, ν_L $(l = e, \mu, \tau)$ are the coherent combination of neutrino mass states ν_i $(i = 1, 2, 3)$ as given by the following relation

$$\nu_l = U_{\mathrm{PMNS}} \nu_i, \tag{1}$$

where U_{PMNS} is the unitary matrix called Pontecorvo-Maki-Nakagawa-Sakata mixing matrix defined as

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-\iota\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{\iota\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{\iota\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{\iota\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{\iota\delta} & c_{23}c_{13} \end{pmatrix}. \tag{2}$$

Here $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$ $(i, j = 1, 2, 3)$, and δ is the Dirac type CP violating phase. The neutrino mass matrix M_ν is related with the unitary mixing matrix and the complex neutrino masses $m_d = \mathrm{diag}(m_1, m_2 e^{2\iota\alpha}, m_3 e^{2\iota\beta})$ by the relation

$$M_\nu = U^*_{\mathrm{PMNS}} m_d U^T_{\mathrm{PMNS}}, \tag{3}$$

where α and β are the Majorana phases.

Neutrino mixing matrix and the corresponding mass matrix have many special forms based upon flavor symmetries. Harrison, Perkins, and Scott proposed one such type of mixing called Tri-BiMaximal (TBM) mixing:

$$U_{\mathrm{TBM}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\sqrt{\frac{1}{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}. \tag{4}$$

TBM ansatz predicts a vanishing θ_{13}. However, θ_{13} is non zero as measured by the recent experiments: T2K, Daya Bay, RENO and DOUBLE CHOOZ. This leads to the realization that although TBM ansatz is ruled out by the experiments, it can still be used as leading order contribution to the neutrino mass matrix.

In the present paper, we propose two simple textures of M_{magic} that break the $\mu - \tau$ symmetry of TBM neutrino mass matrix but preserve its magic symmetry. These textures can be written as

$$M^i_{\mathrm{magic}} = M_{\mathrm{TBM}} + M'_i, \quad (i = a, b). \tag{5}$$

Here, M_{TBM} is the mass matrix for U_{TBM} with the assumption that its lowest eigen value is zero. The $\mu - \tau$ breaking term M_i in these textures is function of only one complex variable $\eta = ze^{\iota\chi}$. Forms of M_{TBM} and M'_i $(i = a, b)$ studied in the present work for normal hierarchy are:

$$\underbrace{\begin{pmatrix} a & a & a \\ a & a+d & a-d \\ a & a-d & a+d \end{pmatrix}}_{M_{\mathrm{TBM}}}, \underbrace{\begin{pmatrix} 0 & 0 & \eta \\ 0 & 0 & \eta \\ \eta & \eta & -\eta \end{pmatrix}}_{M'_a}, \underbrace{\begin{pmatrix} 0 & \eta & 0 \\ \eta & 0 & 0 \\ 0 & 0 & \eta \end{pmatrix}}_{M'_b}. \tag{6}$$

Here, the magic mass matrix M^i_{magic} has the magic symmetry as the sum of the elements in any offer its columns or rows is identical. A magic mass matrix has a trimaximal eigenvector. The mixing matrix corresponding to such mass matrices have their middle column same as that of U_{TBM} (trimaximal) and can be described in terms of two independent variables

*Electronic address: kjschanney@outlook.com
†Electronic address: sanjeevkumarverma@outlook.in

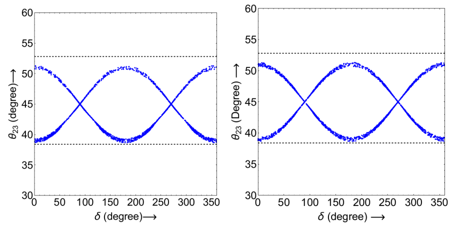

FIG. 1: The correlations between atmospheric angle θ_{23} and CP violating phase δ for both the textures M^a_{magic} and M^b_{magic}.

θ and ϕ:

$$U_{\text{TM}} = \begin{pmatrix} \sqrt{\frac{2}{3}}\cos\theta & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}\sin\theta \\ \frac{e^{i\phi}\sin\theta - \frac{\cos\theta}{\sqrt{3}}}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{-e^{i\phi}\cos\theta - \frac{\sin\theta}{\sqrt{3}}}{\sqrt{2}} \\ \frac{-\frac{\cos\theta}{\sqrt{3}} - e^{i\phi}\sin\theta}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{e^{i\phi}\cos\theta - \frac{\sin\theta}{\sqrt{3}}}{\sqrt{2}} \end{pmatrix}. \tag{7}$$

The mass matrix M^i_{magic} can be diagonalized by using the equation

$$M_d = U^T_{\text{TM}} M_{\text{magic}} U_{\text{TM}}. \tag{8}$$

The diagonal elements of M_d will give us neutrino masses and Majorana phases, whereas the off-diagonal elements, when equated to zero, will give the variables θ and ϕ of U_{TM} in terms of the parameters of M_{magic}. We can calculate the mixing angles and CP violating phase δ in terms of θ and ϕ from the elements of $U = U_{\text{TM}}$ using the relations

$$\sin^2\theta_{23} = \frac{|(U_i)_{23}|^2}{1-|(U_i)_{13}|^2}, \quad J = Im[U_{12}U_{23}U^*_{13}U^*_{22}] \tag{9}$$

Neutrino masses and Majorana phases can be calculated from M_d using the following relations

$$|m_1| = |[M_d]_{11}|, |m_2| = |[M_d]_{22}|, |m_3| = |[M_d]_{33}|, \tag{10}$$

$$\alpha = \frac{1}{2}\arg\left(\frac{[M_d]_{22}}{[M_d]_{11}}\right), \quad \beta = \frac{1}{2}\arg\left(\frac{[M_d]_{33}}{[M_d]_{11}}\right). \tag{11}$$

For details of these calculations, see Ref. [1].

We can diagonalize these mass matrices by using Eq. (8) and obtain our predictions from Eqs. (9-11). Equating the nondiagonal entry $[m_d]_{13}$ with zero for these textures will give us predictions for the variables θ and ϕ in terms of a, d, z and χ.

TABLE I: Allowed ranges of the parameters of mass matrix.

Parameters	Allowed 3σ range	
	M^a_{magic}	M^b_{magic}
a	[-0.008,0.008]	[-0.008,0.008]
d	[0.016,0.034]∪ [-0.034,-0.016]	[0.021,0.027] ∪ [-0.027,-0.021]
z	[0.009,0.013]	[0.009,0.013]

The neutrino mass matrix textures studied in the present work (Eqs. (5,6)) are functions of four variables: a, d, z, and χ. We perform a Monte Carlo analysis for these two textures by generating these variables using uniform random distributions. The varriables a, d, z and χ are constrained by imposing the experimental constraints on $\Delta m^2_{12} = m^2_2 - m^2_1$, $\Delta m^2_{23} = m^2_3 - m^2_2$, θ_{12} and θ_{23} at 3σ C. L (see Table. I). The correlation plots between θ_{23} and δ are shown in Fig. 1 for both the textures. Here, θ_{23} is well within the experimental range ($[38.4, 52.8]$, shown as horizontal dashed lines) for both the textures. We find that for both the textures δ should be either 90^o or 270^o for a maximal θ_{23}. The value of δ shifts towards 0^o or 180^o when θ_{23} takes its extreme values around 38^o or 51^o. This feature is testable at the experiments like NOνA and T2K.

References

[1] K. S. Channey and S. Kumar. Phenomenological implications of two simple modifications to Tri-Bimaximal mixing. *Modern Physics Letters A*, 32(26):1750137, 2017.

[2] P. Harrison and W. Scott. The simplest neutrino mass matrix. *Phys. Lett. B*, 594(3-4):324–332, 2004.