Uncertainty propagation in efficiency calculation of HPGe detector using Unscented Transform method

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Introduction

In nuclear science, we mostly use first order sensitivity analysis methods to study uncertainties in experimental data. The values of physical quantities which cannot be directly measured have to be calculated from variables that can be directly measured based on their functional relation with each other. This method works well for linear functions or the functions with small non-linearities. But there are many problems that involve high order nonlinear relationships between dependent and independent variables. Unscented Transform method can be a better way to handle such non-linearities, because it includes higher order terms of the Taylor series expansion [1]. First we will discuss the first order sensitivity analysis method also known as Sandwich method for error propagation.

First order sensitivity analysis

Let x be an independent vector variable of order n and y be dependent vector variable of order m. Let $y=f(x)$ then the mean value of y is given as $\overline{y} \approx f(\overline{x})$ and the covariance matrix using Sandwich formula is given by

$$C_y \approx H_x C_x H_x^T. \qquad (1)$$

Here C_x is $n \times n$ covariance matrix of x, C_y is $m \times m$ covariance matrix of y and H_x is sensitivity matrix with elements $H_{xij} = \left(\frac{\partial f_i}{\partial x_j}\right)$, $(i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n)$. This method works quite good for functions with small nonlinearity and small uncertainties but as the nonlinearity increases it produces unsatisfactory results. To overcome this higher order terms of Taylor expansion can be involved in calculations to get more accurate results.

Unscented Transform Method

It is difficult to transform a probability density function(PDF) through a general nonlinear function that is why uncertainty propagation is also difficult. Unscented Transform method (UT) is based on two principles; First, it is easy to perform a nonlinear transformation on a single point. Second, it is easy to find a set of individual points in state space whose sample PDF approximates the true PDF of a state vector [1]. Consider a primary variable vector x with mean $\overline{x}$ and covariance P. If we find a set of deterministic vectors called sigma points whose ensemble mean and covariance are same as that of x. Then using these sigma points, on the known nonlinear functional relationship to obtain transformed vectors, we can calculate mean and covariance of transformed vectors.

Let x be an $n \times 1$ vector. We choose $2n$ sigma points $x^{(i)}$ as follow

$$x^{(i)} = \overline{x} + \widetilde{x}^{(i)}, \quad i = 1, 2, \ldots, 2n \qquad (2)$$

where

$$\widetilde{x}^{(i)} = \left(\sqrt{nP}\right)_i^T, \quad \text{and} \quad \widetilde{x}^{(n+i)} = -\left(\sqrt{nP}\right)_i^T, \qquad (3)$$

for $i = 1, 2, \ldots, n$. $\sqrt{nP}$ can be calculated using Cholesky factorization. $2n$ transformed vectors (y) are than calculated using these sigma points. The mean and covariance are

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given by the formula

$$\overline{y} = \sum_{i=1}^{2n} W^{(i)} y^{(i)} \quad (4)$$

$$C = \sum_{i=1}^{2n} W^{(i)} \left(y^{(i)} - \overline{y}\right)\left(y^{(i)} - \overline{y}\right)^T \quad (5)$$

$W^{(i)}$ are are weighting coefficients and defines as $W^{(i)} = 1/2n,\ i = 1, 2, \ldots, 2n$ [1]

Uncertainty propagation inefficiency calculation of HPGe detector

In this experiment the efficiency of the detector is determined at six different energies of the calibration source ^{152}Eu [4]. The number of counts (C) and gamma abundances (a) given in Table I are taken from [4]. The activity (A_0) of the source at the time of its manufacturing was 7767.67± 155.35. And the time elapsed (t) between manufacturing and the experiment date was 9.893 years, half-life (T) of ^{152}Eu is 13.537±0.006 years. The efficiency (ϵ) is given as [4]:

$$\epsilon = \frac{C}{\alpha A_0 e^{\left(\frac{-0.693}{T} t\right)}} \quad (6)$$

In first order sensitivity analysis method the covariance matrix was calculated considering different attributes of the efficiency, i.e., C, a, A_0 and T. The different attributes were considered independent of each other. We have also calculated the efficiency, their standard deviations and covariance using Unscented Transform method. For this we generated 2×14 = 28 sigma points as we had six counts, six gamma abundances, source activity and half-life (14 attributes) in this example.

Result and Discussion

The efficiencies calculated using UT method includes higher-order terms of the Taylor series expansion of the efficiency function, so these values are more accurate than the values calculated using sandwich formula.

TABLE I: Efficiency (ϵ) and their standard deviations ($\Delta\epsilon$) by using Sandwich and Unscented Transform method.

E_γ (keV)	S A Method $\epsilon(\Delta\epsilon)(\times 10^{-2})$	UT Method $\epsilon(\Delta\epsilon)(\times 10^{-2})$
244.675	3.3262(0.0903)	3.3275(0.0906)
411.116	1.9954(0.1236)	1.9962(0.1237)
867.378	0.9042(0.0563)	0.9046(0.0563)
964.079	0.8563(0.0236)	0.8567(0.0238)
1112.074	0.7817(0.0220)	0.7820(0.0221)
1299.140	0.7459(0.0676)	0.7462(0.0676)

TABLE II: Calculated covariance matrix using Sandwich method and Unscented Transform method ($\times 10^{-7}$)

Covariance	8.145					
matrix	2.655	15.266				
using	1.203	0.722	3.165			
Sandwich	1.139	0.684	0.31	0.559		
method	1.04	0.624	0.283	0.268	0.486	
	0.992	0.595	0.27	0.256	0.233	4.567
Covariance	8.218					
matrix	2.699	15.292				
using	1.223	0.734	3.171			
Unscented	1.158	0.695	0.315	0.564		
Transform	1.057	0.634	0.287	0.272	0.490	
method	1.009	0.605	0.274	0.260	0.237	4.571

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