

Comparative Study of Properties of Stable (⁴He) and Weakly Bound (⁶He, ⁸He) Helium Isotopes using Skyrme Pairing Force-SKP and Modified SKM* Functionals

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In last few decades, the ground state properties of nuclei have been studied within the self-consistent mean-field approximation. Such a description of the atomic nucleus has ability to properly account for the bulk properties of nuclei such as masses, energies, radii or shape. The Hartree-Fock method provides a good approximation of closed shell magic nuclei, however, the pairing correlations constitute an essential ingredient for the description of open shell nuclei. These effects are usually described by the Hartree-Fock plus BCS (HFBCS) or Hartree-Fock-Bogolyubov (HFB) methods [1-4]. The HFB problem in coordinate basis, along with the use of the effective Skyrme force is indeed a powerful tool for studying ground-state properties of nuclei.

Another microscopic approach that is well suited in providing quantified predictions throughout the nuclear chart is nuclear Density Functional Theory (DFT) [5-7]. In Density Functional Theory (DFT), the basic idea is that the ground-state energy of a stationary many-body system can be represented in terms of the ground state density alone. Since the density is only a function with three spatial coordinates, DFT calculations are comparatively simple to implement and often very accurate and computationally feasible even for systems with large particle numbers.

An effective interaction in DFT is given by the energy density functional (EDF), whose coupling constants are adjusted to reproduce measured observables. In particular, the Hartree-Fock method with Skyrme interaction becomes the most widely utilized approach to analyse the nuclear structure and related properties.

In its original form Skyrme's interaction can be written as a potential [2].

$$V = \sum_{i < j} v_{ij}^{(2)} + \sum_{i < j < k} v_{ijk}^{(3)}$$

with, $v_{ij}^{(2)}$ is a two body term and $v_{ijk}^{(3)}$ is a three body term. The two body term and three body term were modified by different authors by fitting the large set of experimental data available in literature [2-5].

In this approach, the total Hamiltonian H_T can be expressed as the integral of the density functional [1-4] as given below.

$$H_T = \langle \Psi | H | \Psi \rangle = \int H d^3r$$

with,

$$H = H_{kin} + H_0 + H_{density} + H_{eff} + H_{fin} + H_{so} + H_{sg} + H_{Coul}$$

and various terms have their usual meanings. Besides these, pairing correlations have been known to influence nuclear structure and reaction dynamics of spherical and deformed nuclei and hence must be entertained in the theoretical description. Gogny and his collaborators [8] within the framework of Hartree-Fock Bogolyubov (HFB) theory developed an effective interaction appropriate for description of the mean field and pairing correlations. The Hartree-Fock method with effective Skyrme interaction, wherein one can work in coordinate space and properly handle the particle continuum states in nuclei close to drip lines, is another simple alternative way to include the pairing effects.

	Nucleus	⁴ He	⁶ He	⁸ He
Nuclear Property	Functional			
Radius (fm)				
Proton RMS Radius	SKP	1.956	1.972	2.021
	SKM*	1.946	1.948	1.989
Neutron RMS Radius	SKP	1.948	2.389	2.737
	SKM*	1.938	2.356	2.663
Total RMS Radius	SKP	1.952	2.259	2.577
	SKM*	1.942	2.228	2.512
Charge Radius	SKP	2.124	2.112	2.131
	SKM*	2.104	2.105	2.144
Neutron Skin	SKP	0.000	0.417	0.715
	SKM*	0.000	0.408	0.673
Pairing Energy (MeV)				
Binding Energy	SKP	-29.983	-36.159	-37.137
	SKM*	-30.019	-37.188	-39.728
Proton Fermi Energy	SKP	-10.558	-18.715	-24.327
	SKM*	-10.634	-18.274	-23.621
Neutron Fermi Energy	SKP	-11.478	-4.272	-1.157
	SKM*	-11.580	-4.959	-2.009
Proton Pairing Gap	SKP	5.597	5.015	4.718
	SKM*	5.880	5.416	5.134
Neutron Pairing Gap	SKP	5.649	3.725	3.265
	SKM*	5.956	3.842	3.396

Table 1: Comparison of stable ⁴He and weakly bound ⁶He and ⁸He nuclei properties.

Thus in the present work, we have used the Skyrme Pairing (SkP) and modified SKM* functionals to study properties of stable and weakly bound isotopes of He. The results of calculations are listed in Table1.

By comparing the different properties in Table1, the following meaningful and important conclusions can be drawn. For weakly bound ⁶He and ⁸He nucleus, the neutron r.m.s. radius as well as the total r.m.s. radius is much larger than its stable counterpart ⁴He and confirms the fact that ⁶He and ⁸He have neutron halo structure. In addition, the pairing energies of protons and neutrons are treated separately for all the three nuclei, since neutron rich nuclei would have higher single particle energy of the last filled neutron than the one for protons. Therefore, the

pairing gap decreases with increase of neutron excess which can be seen from neutron pairing gap value of ⁶He and ⁸He (see Table1). Due to excess neutrons and smaller neutron pairing gap ⁶He and ⁸He have neutron halo structure. Furthermore, it is worth mentioning here that both density functionals SKM* and SKP predict the same weakly bound structure for ⁶He and ⁸He. In future our efforts will be focussed to use nuclear energy density functional to unravel various structural properties of weakly bound stable and unstable nuclei and hence to understand the reaction dynamics for reactions induced by these nuclei.

References

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