

## Shell model results with $NN + NNN$ interaction for Ca isotopes and role of $d_{5/2}$ orbital

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### Introduction

The many-body perturbation theory (MBPT) with three nucleon forces ( $3N$ ) is very important to explain the spectroscopy of neutron rich Ca isotopes [1]. In addition, the *ab-initio* calculations with other modern approaches: in-medium similarity renormalization group (IM-SRG) and coupled-cluster effective interaction (CCEI) with chiral NN and  $3N$  forces among valence nucleons are found to give a good description of location of dripline, spectroscopy, radii and other nuclear observables.

In calcium isotopes, closed proton shell has particular attraction for investigating the shell formation. Using phenomenological interaction it was possible to reproduce magic numbers  $N = 2, 8, 20$  very well, but it was not possible to reproduce correctly doubly-magic charter of  $^{48}\text{Ca}$ . Thus, phenomenological interactions have been readjusted which may be due to neglecting three nucleon ( $3N$ ) forces [2]. The  $3N$  forces are very crucial in explaining spectroscopy of Ca chain. In the present work, we have performed shell model calculations with  $NN$  and  $NN + NNN$  interactions and reported results of  $^{49}\text{Ca}$  and  $^{50}\text{Ca}$  isotopes.

### Theoretical Framework

In this work, we have performed shell model calculations in  $pf$ ,  $pf g_{9/2}$  and  $pf g_{9/2} d_{5/2}$  model spaces. To diagonalize the matrices, the shell model codes ANTOINE, KSHELL and NuShellX have been used. For  $pf$  shell phenomenological KB3G interaction which is obtained on the basis of Kuo-Brown's G-matrix

interaction is used. For  $pf g_{9/2}$  space, we have used  $fp g$  interaction [3] and for extended valence space  $pf g_{9/2} d_{5/2}$  GXPF1Br interaction [4] is used. The IM-SRG approach is used for  $NN + NNN$  interaction.

Stroberg *et al.* [5] derived a nucleus-dependent valence-space approach using the IM-SRG, which is normal ordered with respect to a finite-density reference state  $|\Phi\rangle$ . This approach adopts a decoupled valence space Hamiltonian in which occupied orbits are fractionalized.

We can express the effective Hamiltonian in terms of single-particle energies and two and three-body matrix elements, as:

$$H = E_0 + \sum_{ij} f_{ij} \{a_i^\dagger a_j\} + \frac{1}{4} \sum_{ijkl} \Gamma_{ijkl} \{a_i^\dagger a_j^\dagger a_l a_k\} + \frac{1}{36} \sum_{ijklmn} W_{ijklmn} \{a_i^\dagger a_j^\dagger a_k^\dagger a_n a_m a_l\}, \quad (1)$$

where,  $E_0, f_{ij}, \Gamma_{ijkl}$  and  $W_{ijklmn}$  are the normal ordered zero-, one-, two-, and three-body terms, respectively. The normal ordered strings of creation and annihilation operators obey  $\langle \Phi | \{a_i^\dagger \dots a_j\} | \Phi \rangle = 0$ .

Shell model calculations show that  $fp g$  valence space can reproduce reasonable spectra up to  $N \leq 35$  but fails for nuclei around  $N = 40$ , because these nuclei are showing collectivity. Thus to reproduce quadrupole collectivity,  $\nu 1d_{5/2}$  orbital has included in our model space. This can be described in terms of quasi-SU3 symmetry [6]. In the present work we have taken  $pf g_{9/2} d_{5/2}$  model space to know the importance of  $\nu 0g_{9/2}$  orbital for neutron-rich Ca chain.

### Results and Discussions

The comparison of energy spectra with different interactions and experimental data are

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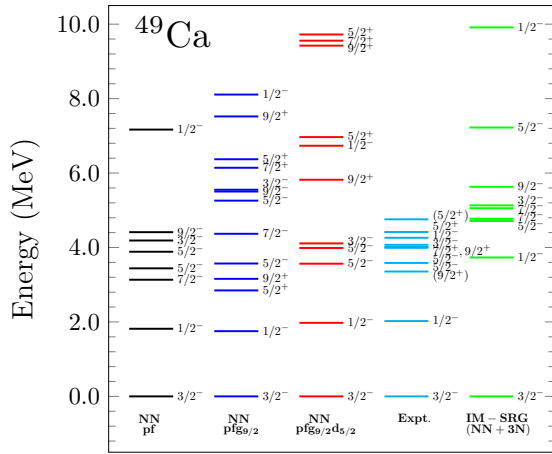


FIG. 1: Comparison between calculated and experimental [7] energy levels for  $^{49}\text{Ca}$ .

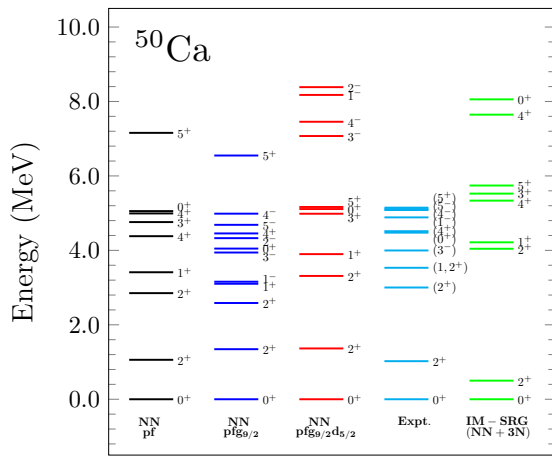


FIG. 2: Comparison between calculated and experimental [7] energy levels for  $^{50}\text{Ca}$ .

shown for  $^{49}\text{Ca}$  and  $^{50}\text{Ca}$  in Figs. 1 and 2, respectively. For  $^{49}\text{Ca}$ ,  $NN$  forces give good

result for first excited state with respect to experimental data. The ground state in  $^{49}\text{Ca}$  is dominated by the single-particle of  $p_{3/2}$  on top of  $^{48}\text{Ca}$ . Therefore, the first excited state  $1/2_1^-$  predicted is in very good agreement with experiment. For other excited states,  $NN$ -forces with  $pf g_{9/2} d_{5/2}$  valence space produce reasonable energy spectra compared to the experimental data. The calculated  $1/2^-$  level is compressed with  $fp g$  interaction. Once we include  $3N$  force (for e.g. in IM-SRG), it gives first excited state at very high energy (it lies  $\sim 1.7$  MeV higher).

For  $^{50}\text{Ca}$ , the location of the first excited state  $2_1^+$  from all the calculations have been predicted very well with experimental data. IM-SRG calculations with  $NN+3N$  forces has under predicted the first excited state by 500 keV. The most of the experimental levels are tentative in case of  $^{50}\text{Ca}$ . The large gap between the  $2_1^+$  and  $2_2^+$  states is reproduced by all phenomenological interactions. The spin of the third excited state has not been experimentally identified, but our calculations predict it as  $1^+$ .

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