Local gauge invariance of Dyon

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Introduction

Dyon is a hypothetical particle in high energy physics that carry both electric and magnetic charge. A dyon with zero electric charge is usually referred to as a magnetic monopole. Julia-Zee showed that for a dyon, a non-abelian gauge theory with Higgs fields exhibits classical solution containing both electric and magnetic charge [1]. In this paper, the manifestly dual symmetric, and local gauge invariance of dyon is developed. The corresponding field equations of dyon are also derived.

Local gauge invariance of dyon

Free particle Dirac equation for dyon [2] is,

$$\begin{aligned} L = i\hbar c\bar{\psi_1}\gamma^\mu\partial_\mu\psi_1 - mc^2\bar{\psi_1}\psi_1 \\ +i\hbar c\bar{\psi_2}\gamma^\mu\partial_\mu\psi_2 - mc^2\bar{\psi_2}\psi_2 \end{aligned} \quad (1)$$

Here ψ_1 is the Dirac spinor [2] for a electric charge, while the other spinor ψ_2 is the Dirac iso-spinors acting on the magnetic monopole. This Dirac Lagrangian for dyon is invariant under the global gauge transformation as,

$$\psi_1 \longrightarrow e^{i\theta_1}\psi_1 \quad (2)$$

$$\psi_2 \longrightarrow e^{i\theta_2}\psi_2 \quad (3)$$

Here, θ is any real number. In local gauge transformation, the phase factor is different at different space-time point, *i.e.*, θ is a function of the coordinates of the points in space-time:

$$\psi_1 \longrightarrow e^{i\theta_1(x_1)}\psi_1 \quad (4)$$

$$\psi_2 \longrightarrow e^{i\theta_2(x_2)}\psi_2 \quad (5)$$

The Lagrangian is not invariant under such a local gauge transformation. An extra term from the derivative of θ is taken up to make it symmetric.

$$\partial_\mu(e^{i\theta_1}\psi_1) = i(\partial_\mu\theta_1)e^{i\theta_1}\psi_1 + e^{i\theta_1}\partial_\mu\psi_1 \quad (6)$$

$$\partial_\mu(e^{i\theta_2}\psi_2) = i(\partial_\mu\theta_2)e^{i\theta_2}\psi_2 + e^{i\theta_2}\partial_\mu\psi_2 \quad (7)$$

so that

$$L \to L - \hbar c\,(\partial_\mu\theta_1)\,\bar{\psi_1}\gamma^\mu\psi_1 \quad (8)$$

$$L \to L - \hbar c\,(\partial_\mu\theta_2)\,\bar{\psi_2}\gamma^\mu\psi_2 \quad (9)$$

To pull a factor of $\frac{-e}{\hbar c}$ and $\frac{-g}{\hbar c}$ out of θ_1 and θ_2, letting

$$\lambda(x_1) = \frac{-\hbar c}{e}\theta_1(x_1) \quad (10)$$

$$\lambda(x_2) = \frac{-\hbar c}{g}\theta_2(x_2) \quad (11)$$

where e and g are the charges of the particle in term of λ, then

$$L \to L + (e\bar{\psi_1}\gamma^\mu\psi_1)\partial_\mu\lambda \quad (12)$$

$$L \to L + (g\bar{\psi_2}\gamma^\mu\psi_2)\partial_\mu\lambda \quad (13)$$

Under local gauge transformation as,

$$\psi_1 \to e^{\frac{-ie\lambda(x_1)}{\hbar c}}\psi_1 \quad (14)$$

$$\psi_2 \to e^{\frac{-ig\lambda(x_2)}{\hbar c}}\psi_2 \quad (15)$$

To convert free Dirac Lagrangian into local gauge invariance, few terms added to pick up the extra term in equations (12) and (13) as,

$$\begin{aligned} L = i\hbar c\bar{\psi_1}\gamma^\mu\partial_\mu\psi_1 - mc^2\bar{\psi_1}\psi_1 + i\hbar c\bar{\psi_2}\gamma^\mu\partial_\mu\psi_2 \\ -mc^2\bar{\psi_2}\psi_2 - e\bar{\psi_1}\gamma^\mu\psi_1A_\mu - g\bar{\psi_2}\gamma^\mu\psi_2B_\mu \end{aligned} \quad (16)$$

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A_μ and B_μ transforms under local gauge transformation as,

$$A_\mu \rightarrow A_\mu + \partial_\mu \lambda \tag{17}$$

$$B_\mu \rightarrow B_\mu + \partial_\mu \lambda \tag{18}$$

The Lagrangian [eq.(16)] is invariant under a local gauge transformation, but eq.(16) is not the complete Lagrangian, the full Lagrangian must include a free term for the gauge field. Since it is a vector field, so we consider proca Lagrangian for dyon [3] as,

$$L = -\frac{1}{4}F_{\mu\nu}\ F^{\mu\nu} - \frac{1}{4}M_{\mu\nu}M^{\mu\nu} + \frac{1}{2}\mu^2 A_\mu A^\mu + \frac{1}{2}\mu^2 B_\mu B^\mu \tag{19}$$

Here, $F^{\mu\nu}$ and $M^{\mu\nu}$ are the invariants under equations (17) and (18) while $A_\mu A^\mu$ and $B_\mu B^\mu$ is not an invariant, The gauge field must be massless ($\mu = 0$) otherwise local gauge invariance will be lost. To start with Dirac Lagrangia, local gauge invariance is imposed and a massless vector fields A^μ and B^μ are introduced which resulted into the complete Lagrangian *i.e.*

$$\begin{aligned} L = i\hbar c\bar{\psi_1}\gamma^\mu\partial_\mu\psi_1 - mc^2\bar{\psi_1}\psi_1 + i\hbar c\bar{\psi_2}\gamma^\mu\partial_\mu\psi_2 \\ -mc^2\bar{\psi_2}\psi_2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}M_{\mu\nu}M^{\mu\nu} \\ -(e\bar{\psi_1}\gamma^\mu\psi_1)A_\mu - (g\bar{\psi_2}\gamma^\mu\psi_2)B_\mu \end{aligned} \tag{20}$$

The last four-terms in equation (20) reproduce the Maxwell Lagrangian of dyon with the current density.

$$j^{\mu(e)} = e(\bar{\psi_1}\gamma^\mu\psi_1) \tag{21}$$

$$j^{\mu(g)} = g(\bar{\psi_2}\gamma^\mu\psi_2) \tag{22}$$

The condition of local gauge invariance of dyon specifies the current produced by Dirac particles in free Dirac Lagrangian of dyon. The difference between global and local transformation arises by calculating derivatives of the fields equations(6)and (7).

$$\partial_\mu\psi_1 = e^{\frac{-ie\lambda}{\hbar c}}[\partial\mu - i\frac{e}{\hbar c}(\partial_\mu\lambda)]\psi_1 \tag{23}$$

$$\partial_\mu\psi_2 = e^{\frac{-ig\lambda}{\hbar c}}[\partial\mu - i\frac{g}{\hbar c}(\partial_\mu\lambda)]\psi_2 \tag{24}$$

In the original(free)Lagrangian,every derivative (∂_μ) into the covariant derivative is replaced for electric and magnetic charge as,

$$\mathcal{D}_\mu = \partial_\mu + i\frac{e}{\hbar c}A_\mu \tag{25}$$

$$\nabla\mu = \partial_\mu + i\frac{g}{\hbar c}B_\mu \tag{26}$$

The transformation of A_μ and B_μ [equations (17) and (18)] will cancel the offending term in eq. (23) and (24).

$$\mathcal{D}_\mu\psi_1 \rightarrow e^{\frac{-ie\lambda}{\hbar c}}\mathcal{D}_\mu\psi_1 \tag{27}$$

$$\nabla_\mu\psi_2 \rightarrow e^{\frac{-ig\lambda}{\hbar c}}\nabla_\mu\psi_2 \tag{28}$$

The substitution of $\mathcal{D}_\mu$ and ∇_μ in place of ∂_μ for electric and magnetic charge respectively, is a simple method for converting a globally invariant Lagrangian into the locally invariant. The Lagrangian eq.(20) is considered to get the Maxwell field equation for dyon as,

$$\vec{F}_{\mu\nu,\nu} = j_\mu^{(e)} \tag{29}$$

$$\vec{M}_{\mu\nu,\nu} = j_\mu^{(g)} \tag{30}$$

Conclusion

This paper derives a simple method for converting a globally invariant Lagrangian into a locally invariant for dyon. It shows the global phase transition eqations (2) and (3) as multiplication of ψ_1 and ψ_2 by a unitary matrix ($U^{(e)} = e^{i\theta_1}$,($U^{(g)} = e^{i\theta_2}$) for electric and magnetic charge respectively. The group of all such matrices are $U^{(e)}(1) \times U^{(g)}(1)$, and the symmetry involved is called $U^{(e)}(1) \times U^{(g)}(1)$ gauge invariance of dyon.

References

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