

# Semiclassical description of $\alpha$ radioactivity in spherical nuclei

Saniya Monga<sup>1</sup>, Harjeet Kaur<sup>1</sup>

<sup>1</sup> Department of Physics, Guru Nanak Dev University, Amritsar-143005  
saniyamonga3994@gmail.com

## Introduction

The level density can be written as a sum of average and oscillating part,  $g(E) = \tilde{g}(E) + \delta g(E)$ . As a result of the connection between oscillating part of level density and the classical periodic orbits, shell effects appear in the level density. Utilizing the level density for spherical harmonic oscillator along with spin-orbit interactions [1], the present work highlights the description of  $\alpha$  radioactivity in spherical nuclei such as Sm<sup>146</sup>, Gd<sup>148</sup>.  $Q_\alpha$  values for these are calculated by introducing shell and pairing corrections to the liquid drop binding energy.

## Methodology

Among all the possible decay modes of heavy nuclei,  $\alpha$  decay is the most important, because it provides detailed information on nuclear structure, nuclear interaction as well as in the identification of new elements. In this work, we would like to introduce shell structures  $\delta U$  and pairing correlations  $\delta P$  in the binding energy (function of atomic number  $Z$  and mass number  $A$ ) calculation:

$$B(Z, A) = B_{LDM}(Z, A) + \delta U + \delta P. \quad (1)$$

Here,  $B_{LDM}(Z, A)$  [2] is given as:

$$B_{LDM}(Z, A) = a_v A - a_s A^{\frac{2}{3}} - a_c \frac{Z^2}{A^{\frac{1}{3}}} - a_a \frac{(N - Z)^2}{A}.$$

### (1) Introducing the shell corrections

The consideration of shell structures is important to incorporate the details of the nuclear

structure in terms of the appropriate shell closures. These can be taken care of through the total and only the smooth part of the level density [1] and can be calculated as:

$$\delta U(n, p) = 2 \int_0^{E'_F} E g_{n,p}(E) dE - 2 \int_0^{E_F} E \tilde{g}_{n,p}(E) dE \quad (2)$$

where,  $E'_F$  and  $E_F$  represents the Fermi energy, with and without shell effects respectively.

**(2) Introducing the pairing corrections**  
The tendency of like nucleons to pair up greatly affects the nuclear structure especially at low energies. Nuclear superfluid transitions are quite well known. The pairing correlation energy can be defined as:

$\delta P_{n,p} = |\tilde{E}_{pair} - \tilde{E}|$  where

$$\begin{aligned} \tilde{E}_{pair}(n, p) &= 2 \int_{-\infty}^r E \tilde{g}_{n,p}(E) dE - \frac{\Delta_{n,p}^2}{G_{n,p}} \\ &+ \int_r^s E \tilde{g}_{n,p}(E) \left[ 1 - \frac{E - \lambda'_{n,p}}{E_q^{n,p}} \right] dE \\ \tilde{E}(n, p) &= 2 \int_{-\infty}^{E_F^{n,p}} E \tilde{g}_{n,p}(E) dE. \end{aligned} \quad (3)$$

Here,  $r, s = \lambda'_{n,p} \mp \hbar\omega_{n,p}$ .  $\lambda'_{n,p}$  and  $E_F^{n,p}$  are the chemical potentials with and without pairing. The quasiparticle energy is defined as,  $E_q = ((E - \lambda'_{n,p})^2 + \Delta_{n,p}^2)^{1/2}$ .  $\Delta_{n,p}$  are the pairing gaps obtained from the four-point formula. We took 80% of the pairing gaps value for our calculations.  $G_{n,p}$  is the pairing strength assumed constant in the BCS theory. The spacing between the oscillator levels is:

$$\hbar\omega_{n,p} = \frac{41}{A^{\frac{1}{3}}} \left( 1 \pm \frac{N - Z}{A} \right)^{\frac{1}{3}} \text{ MeV}. \quad (4)$$

In the above expressions, the level density for spherical harmonic oscillator along with spin-orbit interactions is used, whose average part is:

$$\begin{aligned} \bar{g}(E) &= \frac{E^2}{2\hbar^3\omega^3} \left[ 1 + 3\kappa^2\hbar^2\omega^2 \right] \\ &- \frac{1}{8\hbar\omega} [1 + 5\kappa^2\hbar^2\omega^2] \\ &+ E\kappa^3\hbar\omega + O(\hbar^4\kappa^4) + \dots \end{aligned} \quad (5)$$

and the corresponding oscillating part employed is as given in [1]. Adding these corrections to liquid drop model  $B_{LDM}(Z, A)$  [2], we get the modified binding energy  $B(Z, A)$  as stated in eq.(1).

### (3) Evaluation of $Q_\alpha$ value

We have considered  $\text{Sm}^{146}$ ,  $\text{Gd}^{148}$  nuclei, which are assumed to be spherical in their ground states. Each of which is unstable and undergoes alpha decay to form  $\text{Nd}^{142}$ ,  $\text{Sm}^{144}$  respectively.  $Q_\alpha$  value or the  $\alpha$  disintegration energy can be used to probe the nuclear structure, as it depends on the nuclear transition from the parent to the daughter state. It can be calculated as:

$$Q_\alpha = B_D(Z, A) + B_\alpha - B_P(Z, A). \quad (6)$$

Here, the binding energies of parent and daughter are evaluated as in eq.(1), while the binding energy of  $\alpha$  particle is taken from [3]. The various structure parameters used for the calculations are shown in table 1.

Nucleus	$\kappa_p$ $(\hbar\omega_p)^{-1}$	$\kappa_n$ $(\hbar\omega_n)^{-1}$	$\Delta_p$ (MeV)	$\Delta_n$ (MeV)	$(k_p, k_n)$
$\text{Sm}^{146}$	-0.065	-0.062	1.08	0.75	(5,9)
$\text{Nd}^{142}$	-0.065	-0.060	0.98	1.10	(5,5)
$\text{Gd}^{148}$	-0.065	-0.062	1.14	0.74	(5,5)
$\text{Sm}^{144}$	-0.065	-0.060	0.99	1.14	(5,5)

Table 1: Spin-orbit strength parameters  $\kappa_{p,n}$ , pairing gaps  $\Delta_{p,n}$  and repetitions over periodic orbits  $(k_p, k_n)$  used for our calculations.

## Results and Discussion

The obtained shell, pairing corrections, binding energies and the  $Q_\alpha$  values are listed in table 2 and 3 respectively.

Nucleus	$\delta U$ (MeV)	$\delta P$ (MeV)	$B(Z, A)$ (MeV)	$B_E(Z, A)$ (MeV)
$\text{Sm}^{146}$	-1.9576	2.0060	1209.27	1210.90
$\text{Nd}^{142}$	-0.4694	2.4744	1183.46	1185.14
$\text{Gd}^{148}$	-0.8275	2.1704	1220.59	1220.75
$\text{Sm}^{144}$	-0.1245	2.6265	1195.08	1195.73

Table 2: The shell and pairing corrections  $\delta U$  and  $\delta P$ , obtained  $B(Z, A)$  and the experimental values  $B_E(Z, A)$  [4].

Parent nucleus	Daughter nucleus	$Q_\alpha$ (MeV)	$Q_\alpha^{Expt.}$ (MeV)
$\text{Sm}^{146}$	$\text{Nd}^{142}$	2.486	2.529
$\text{Gd}^{148}$	$\text{Sm}^{144}$	2.786	3.271

Table 3: The comparison of obtained  $Q_\alpha$  and the corresponding experimental value  $Q_\alpha^{Expt.}$  [4].

It is observed that the calculated  $Q_\alpha$  value matches with the experimental value  $Q_\alpha^{Expt.}$  quite well. This  $Q_\alpha$  value can yield important information about nuclear structure and its stability.

## References

- [1] H. Kaur and S. R. Jain, Journal of Physics G: Nuclear and Particle Physics **42**, 115103 (2015).
- [2] J.L. Basdevant, J. Rich and M. Spiro, Fundamentals in Nuclear Physics. Springer (2005).
- [3] M. Wang, G. Audi, F.G. Kondev, W.J. Huang, S. Naimi and Xing Xu, Chinese Physics C **41**,030003 (2017).
- [4] <https://www.nndc.bnl.gov>.