Clustering in Calcium isotopes

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The excitation energy, E^* is given by the product of energy of each mode, and, the number of excited modes (or states) [1]. The number of modes is given by the well-known Weyl formula [2]. Weyl showed by employing the Green function in three dimensions that the cumulative density of states below a wavenumber k ($(2mE/\hbar^2)^{1/2}$) is given by $Vk^3/6\pi^2 - Sk^2/16\pi$, in an asymptotic expansion in powers of k. Combining Weyl formula with the argument by Weisskopf, it immediately follows that the leading dependence of E^* on temperature will be $\sim T^{5/2}$ with a correction term, $\propto T^2$. V and S denote the volume and surface area of the nucleus. The area and volume of nuclei are considered by using the well accepted relation of the radius of the nucleus as $r \sim r_0 \times A^{1/3}$ fm, where, r_0 is taken as 1.2 fm. This gives us excitation energy per nucleon as

$$\begin{aligned} \frac{E^*}{A} &= \frac{4\sqrt{2}}{9\pi}\frac{r_0^3 M^{3/2}}{(\hbar c)^3}(k_b T)^{5/2} \\ &- \frac{r_0^2}{8}\frac{M}{(\hbar c)^2}A^{-1/3}(k_b T)^2 \end{aligned} \quad (1)$$

Dependence of excitation energy with temperature can be given in terms of level density as [4]

$$\begin{aligned} E^* &= \int_0^\infty Eg(E)\left[1 + exp\left(\frac{E-\mu}{T}\right)\right]^{-1} dE \\ &- \int_0^{\epsilon_f} Eg(E)dE \end{aligned} \quad (2)$$

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We calculate the excitation energy for various temperatures using semiclassical level densities [3] for seven nuclei, namely ^{34}Ca, ^{36}Ca, ^{38}Ca, ^{42}Ca, ^{44}Ca, ^{46}Ca, ^{48}Ca. By using these values of excitation energies at a given temperature, we find the value of M from equation 1. The argument by Weisskopf (and Niels Bohr (see footnote in [1]) models nucleus where a "particle" of mass M is confined in a mean-field of others. It is clear that a nucleon alone will experience strong correlations (pairing being an instance). However, in the manner we have articulated, mass can be inferred from (1). And indeed, the size of the cluster would depend on temperature as the interaction binding the nucleons in a cluster are energy-dependent. For instance, a larger excitation energy would be shared also by a nucleon in a cluster, facilitating easier tunneling path from the cluster to be free. We plot this temperature-dependent mass in Fig. 1. In a way, M gives the effective mass of the representative "particle" in terms of which relation of E^* and T of the nucleus can be logically presented. If nucleus is comprised of clusters of nucleons, then the quantity of M will give the mass of the cluster. Here we plot $\frac{M}{m_n}$ against temperature, where m_n is the mass of the nucleon which is taken to be 938 MeV. This shows how, with an increase in temperature, the mass of an individual cluster changes.

It can be seen that similar calculations with only the area term (it is important to note that this is equivalent to using $E^* \sim T^2$), $\frac{M}{m_n}$ turns out to be about 330 to 350 times the mass of a nucleon. This is an indication that the widely accepted $E^* \propto T^2$, is inappropriate, and in fact, a more careful understanding of

Weisskopf's intuition and Weyl's rigour give $E^* \propto T^{5/2}$, with a T^2 term coming next in the degree of significance.

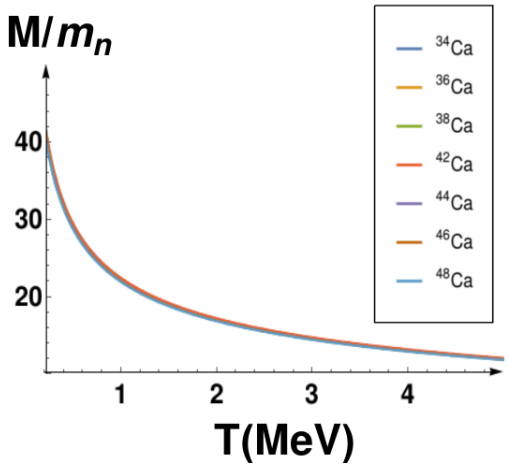

FIG. 1: The change of cluster mass with respect to temperature for various nuclei is plotted by using the semiclassical level density and Weyl formula with volume term and area corrections. It can be seen that at lower energies, all the nucleons behave as a single cluster of number of nucleons around 40 and as the energy increases these clusters break. It also indicates that the widely accepted $E^* \propto T^2$, is inappropriate and the use of $E^* \propto T^{5/2}$, with a correction term of T^2 as the next degree of significance is more proper.

These findings are independent of the stability of isotopes. To summarize, clustering is shown to ensue naturally from the statistical mechanical arguments for excitation energy. Furthermore, the excitation energy follows a five-half exponent on temperature, rather than a quadratic.

References

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