Ignatyuk damping coefficient - from the haze of fitting to an expression

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Introduction

Level density of a nucleus is widely extracted from transmission data [1] or gamma energy spectrum of the nucleus [2]. It is an important experimental quantity which helps in understanding transitions and reaction cross sections. This extraction goes through lot of rigorous model fittings from codes like CASCADE [3]. An excitation energy-dependent form for fitting the level density parameter was proposed by Ignatyuk [4] as:

$$a(E^*) = \tilde{a}\left[1 + \frac{\delta W}{E^*}\{1 - \exp(-\gamma E^*)\}\right] \qquad (1)$$

Here, $\tilde{a}$ is the asymptotic value of nuclear level density parameter, γ is the damping coefficient and δW is the shell correction term which is the difference in the experimental binding energy of nucleus and that calculated from the liquid drop model [9]. This damping parameter has no exact expression and is often employed as a fitting parameter.

In this work we give an exact expression for this damping coefficient, γ, with no adjustable parameters. This is done by using an expression for the level density based on the semiclassical trace formula [5].

Level density parameter and the damping coefficient

For a spherically symmetric harmonic oscillator potential including spin-orbit interactions, finite-temperature single-particle level density $\tilde{g_T}(E)$ has been obtained analytically [5]:

$$\begin{aligned}\tilde{g_T}(E) &= \frac{1+3\kappa^2\hbar^2\omega_0^2}{\hbar^3\omega_0^3}\left(E^2 + \frac{\pi^2T^2}{3}\right)\\ &- \frac{\left(1+5\kappa^2\hbar^2\omega_0^2\right)}{4\hbar\omega_0} + 2\kappa^3\hbar\omega_0 T\ln\left[1+\exp\left(\frac{E}{T}\right)\right]\\ &+ \ldots \end{aligned} \qquad (2)$$

where κ is the spin-orbit interaction strength parameter. For nuclei with $N \neq Z$, the chemical potential $\tilde{\mu}^{(n,p)}$ is fixed from proton and neutron number [6]. If we utilize the total finite-temperature level density $g_T(E)$ [5] to find the chemical potential μ's, then the total level density parameter a_T is given as:

$$a_T = \frac{\pi^2}{6}\left[g_T(\mu^n) + g_T(\mu^p)\right]. \qquad (3)$$

However, the link between the temperature and the excitation energy $E^* = \sum_{n,p} E^*_{n,p}$ for a nucleus can be found by defining it as difference between the internal energies at finite temperature and at zero temperature [6] i.e.

$$\begin{aligned} E^*_{n,p} &= E_{n,p}(T) - E_{n,p}(T=0)\\ &= \int_0^\infty Eg(E)\left[1+\exp\left(\frac{E-\mu}{T}\right)\right]^{-1} dE\\ &- \int_0^{\epsilon_F^{n,p}} Eg(E)dE. \end{aligned}$$

where $g(E)$ is the zero-temperature single-particle level density [7] and $\epsilon_F^{n,p}$ are Fermi energies found using neutron number and proton numbers [6], respectively. Thus, excitation energy dependent level density parameter $a(E^*)$ is found using, $\hbar\omega_0 = \frac{41}{A^{1/3}}$MeV for nuclei with $N = Z = A/2$ and for $N \neq Z$, we have used [8],

$$\hbar\omega_0(n,p) = \hbar\omega_0\left(1 \pm \frac{(N-Z)}{A}\right)^{1/3}. \qquad (4)$$

Also, the spin-orbit interaction strength parameter κ and the repetitions over the classical periodic orbits k are used as given in [5, 8].

Damping coefficient following from (6),

$$\gamma = -\frac{1}{E^*}\log\left[1 + \frac{E^*}{\delta W}\left(1 - \frac{a(E^*)}{\tilde{a}}\right)\right], \qquad (5)$$

is determined and investigated for various nuclei in the present work.

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Results and Discussion

For $^{56,78}Ni$, $^{208,210}Pb$, ^{82}Ge and ^{84}Se nuclei, we find the excitation energy dependent level density parameter and then employing (7), we find damping coefficient γ using the parameters as given in Table I.

Nucleus	δW (MeV)	$\kappa_p(\hbar\omega_p)^{-1}$	$\kappa_n(\hbar\omega_n)^{-1}$	k_p	k_n
^{56}Ni	-5.300	κ =-0.05	k=7 ($N = Z$)		
^{208}Pb	-9.923	-0.151	-0.134	10	9
^{210}Pb	-6.945	-0.151	-0.134	10	9
^{78}Ni	-10.842	-0.075	-0.060	5	5
^{82}Ge	-3.803	-0.090	-0.060	8	5
^{84}Se	-1.880	-0.090	-0.060	8	5

TABLE I: Shell correction term δW, spin-orbit interaction strength parameters κ's and the repetitions over the classical periodic orbits k's used in our calculations are given in this Table.

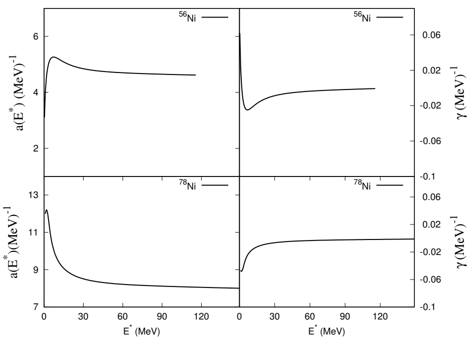

FIG. 1: Excitation energy dependent level density parameter $a(E^*)$ and damping coefficient γ are plotted with respect to the excitation energy for ^{56}Ni and ^{78}Ni nuclei.

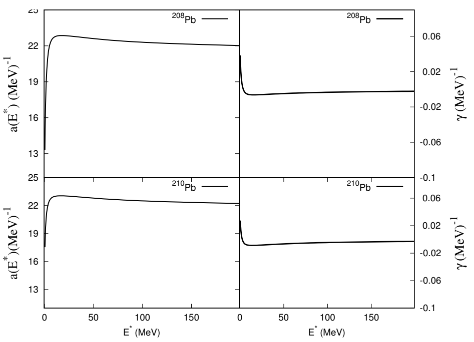

FIG. 2: Excitation energy dependent level density parameter $a(E^*)$ and damping coefficient γ are plotted with respect to the excitation energy for ^{208}Pb and ^{210}Pb nuclei.

It is known [5, 10] that the level density parameter may increase (like in ^{56}Ni and $^{208,210}Pb$) or decrease (like in ^{78}Ni, ^{82}Ge and ^{84}Se) with the excitation energies, until it attains a saturated value. This leads

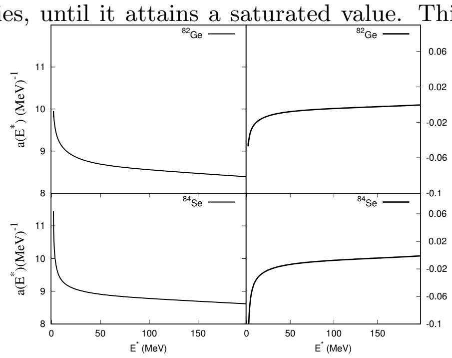

FIG. 3: Excitation energy dependent level density parameter $a(E^*)$ and damping coefficient γ are plotted with respect to the excitation energy for ^{82}Ge and ^{84}Se nuclei.

to both positive and negative values of the damping coefficient until it approaches to zero at saturation of level density. The values of γ obtained here are of the order of experimentally deduced values. Here the values of $\tilde{a}$ are taken from the average semiclassical level density [5], which almost follow a trend of $A/10$. In practice, this behaviour is taken as A/k, where k is a parameter which can vary from 8 to 12 [3]. If we consider this variation of the k parameter as well, the experimental values in the range $0.04 \sim 0.06$ can also be obtained.

References

[1] Md. Abdullah Al Mamun, *Thermal Properties of Nuclei and Their Level Densities*, Masters Thesis, Faculty of the College of Arts and Science, Ohio University, (2015).

[2] P. C. Rout, et al., Physical Review Letters **110**, 062501 (2013).

[3] Pratap Roy et al., Physical Review C **94**, 064607 (2016).

[4] A. V. Ignatyuk, G. N. Smirenkin and A. S. Tishin, Sov. J. Nucl. Phys. **21** 255 (1975).

[5] H. Kaur and S. R. Jain, J. Phys. G **42**, 115 (2015)

[6] M. Brack and R. K. Bhaduri, *Semiclassical Physics*, Westview Press (2003).

[7] B. K. Jennings, R. K. Bhaduri and M. Brack, Nuclear Physics A **253**,29 (1975).

[8] I. Ragnarsson and S. G. Nilsson, *Shapes and Shells in Nuclear Structure*, Cambridge Univ. Press (2005).

[9] https://www.nndc.bnl.gov/

[10] S. Hilaire, Phys. Lett. B **583**, 264 (2004).