

Quasimolecular resonances: a doorway to fusion and oscillatory structure in the second energy derivative of the product of fusion cross section and energy

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Measurements with high precision for the fusion cross sections, σ_f , in the collision of two nuclei reveal that the variation of the results of σ_f with incident energy shows oscillatory structure for the light nuclear pairs namely $\alpha+^{40}\text{Ca}$, $^{12}\text{C}+^{12}\text{C}$, $^{16}\text{O}+^{16}\text{O}$, $^{28}\text{Si}+^{28}\text{Si}$ etc. The same results of σ_f at different energies when extracted in the form of the energy derivative of the product of cross section and energy, $D(E)=\frac{d(E\sigma_f)}{dE}$, the results become more oscillatory as a function of energy. Further, the extracted results of second energy derivative of the cross section times energy, $B(E)=\frac{d^2(E\sigma_f)}{dE^2}$, also show similar oscillatory structure with large number of peaks and deeps sometimes with negative values. Thus, these oscillatory structures in the quantity $B(E)$, so called barrier distribution function, originates from the structure in the results of σ_f which happens to be a part of the total reaction cross sections, σ_r , at a given incident energy. In a collision process, the occurrence of resonances is manifested as peak structure in the variation of σ_r or σ_f as a function of energy. Hence, the presence of oscillations or fluctuations in the results of σ_f clearly indicates that the two-body system has gone through the molecular resonant state before fusion or compound nucleus formation. These results of σ_f with oscillations when presented in the form of $B(E)$ shows oscillatory structure and hence, the root cause for this structure in $B(E)$ can be the molecular resonances occurring in the collision of two nuclei prior to the amalgamation of the pair for a single

compound nucleus. We exhibit this feature of molecular resonance governing fusion reaction by a model calculation for σ_f within the framework of S-matrix theory and explain the experimental data in the case of the $^{28}\text{Si}+^{28}\text{Si}$ system.

The scattering amplitude is written in the usual way as

$$f(\theta) = f_c(\theta) + \frac{i}{2k} \sum_{\ell=0}^{\infty} (2\ell + 1) e^{2i\sigma_\ell} (1 - S_\ell) P_\ell(\cos\theta),$$

where $f_c(\theta)$ is the Rutherford scattering amplitude, σ_ℓ is the Coulomb phase-shift; the S-matrix or the coefficient of reflection S_ℓ is written as

$$S_\ell = S_\ell^0 + S_\ell^r.$$

The quantity S_ℓ^0 is the background scattering matrix element responsible for the forward angle diffraction pattern of the elastic scattering angular distribution, while S_ℓ^r is the resonant term and/or a fluctuating term of statistical origin which corresponds to intermediate structure and/or compound nucleus reactions and is written as

$$S_\ell^r = \frac{r_R}{E - Z_R},$$

where $r_R = \pi\alpha$ is the residue of the S-matrix at the resonance energy $Z_R = \beta\ell$, with α and β two real numbers, and $|S_\ell| \leq 1$ (unitary condition).

It turns out from our analysis that α is small of the order of 0.01 and $\beta=1.5$. The matrix elements S_ℓ^0 are given by a Woods-Saxon form using a semiclassical parametrization [1]

$$S_\ell^0 = \{1 + \exp[(\ell_g - \ell)/\Delta]\}^{-1} + i\mu \frac{d}{d\ell} \{1 + \exp[(\ell_g - \ell)/\Delta]\}^{-1}.$$

The grazing wave ℓ_g and angular momentum width Δ are given by the semiclassical formulas

$$\ell_g = kR \left[1 - \frac{2\eta}{kR}\right]^{1/2},$$

$$\Delta = kd \left[1 - \frac{\eta}{kR}\right] \left[1 - \frac{2\eta}{kR}\right]^{-1/2},$$

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where η and k are, respectively, the Sommerfeld parameter and wave number, and

$$R = r_0(A_T^{1/3} + A_P^{1/3}).$$

It is well known that r_0 is of the order of 1.45 fm for alpha scattering. The parameter d is the surface diffusivity of the target nucleus. Therefore, the background matrix elements depend only on three parameters, r_0 , d , and μ , which can be determined by fitting the elastic scattering angular distributions at very forward angles. The Woods-Saxon parametrisation of these background scattering matrix elements S_ℓ^0 tells us that we are starting our analysis with a strong absorption model since the S_ℓ^0 terms vanish for small ℓ values.

The analysis is characterized by the number of fluctuating terms S_ℓ^r due to overlapping resonances in the compound system, i.e., intermediate quartet structure and/or statistical compound nucleus resonances.

The fusion cross section is given by the total reaction cross section obtained by using the modulus of the total scattering matrix element $S_\ell = S_\ell^0 + S_\ell^r$ in the formula

$$\sigma_f = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell + 1) (1 - |S_\ell|^2).$$

We apply our calculation to the case of $^{28}\text{Si} + ^{28}\text{Si}$ system.

The values of the parameters used are $r_0 = 1.48$ fm, $\mu = 1$, $d = 0.3$, $\alpha = 0.01$, and $\beta = 1.5$. In fig. 1 (a) we plot our results of σ_f as a function of incident energy by a solid curve where the corresponding results from experiment are shown by open circles. It is seen that our calculated results (solid curve) account for the small oscillatory structure seen in the measured data. In the same figure 1 in (b) we compare our calculated results (solid curve) of $D(E) = \frac{d(E\sigma_f)}{dE}$ with the corresponding measured data shown by solid circles and in (c) measured data (solid circles) of $B(E) = \frac{d^2(E\sigma_f)}{dE^2}$, are compared with our calculated results shown by a solid curve.

We can very well see that the oscillatory structure found both in the data of the quantities $D(E)$ and $B(E)$ are explained satisfactorily by our calculated results. Hence, we confirm that the structure with numerous peaks and deeps in the data of $B(E)$ is due to os-

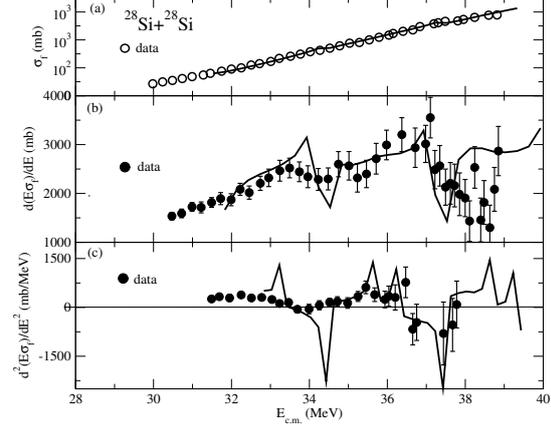


FIG. 1: Plots of σ_f , $D(E) = \frac{d(E\sigma_f)}{dE}$, and $B(E) = \frac{d^2(E\sigma_f)}{dE^2}$ in (a), (b), and (c), respectively, as a function of incident energy. The calculated results are shown by solid curves and the open and solid circles represent respective measured data obtained from [2].

cillatory nature in fusion cross section which represents the quasimolecular resonances exhibited by the bombarding pair of nuclei before proceeding to form a solitary compound body.

References

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