The α-decay half-life from resonance pole of S-matrix of a solvable potential

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α-decay in nuclear physics (NP) studies remains of considerable topical interest specially in the production and study of very heavy nuclei in the actinide and superheavy regions. Recently, five new unstable isotopes in the actinide region have been reported. We present, here, the results of the calculations of half lives of these isotopes and the nuclides appearing in their α-decay chains. As an illustration, we present and discuss the results of the decay ^{217}Ac(Z=89)$\rightarrow$ α+^{213}Fr(Z=87). Microscopic calculations usually are carried out in the WKB framework which requires the Q-values and the α+daughter potential. Due to high sensitivity of the calculated half lives on the Q-values used, usually the experimental Q-values are employed. Several phenomenological expressions based on the Viola-Seaborg formula are also available in the literature [1].

The α+daughter potential can be calculated by folding the effective nucleon-nucleon interaction with the density distributions of α and that of the daughter nucleus (double folding model or t$\rho\rho$ approximation)[2]. Recently, calculations based on the S-matrix formulation have been reported [3]. There, the Schrödinger equation is solved with exact solvable potential which mimics the α+daughter potential calculated in the double folding model. The resonance poles of S-matrix in the complex k-plane then yield the Q-values and half lives.

In the present S-matrix calculation, we use the following exactly solvable α+daughter potential:[4]

$$V_{eff}(r) = H_0 \left\{ 1 - [\frac{1-e^{-(r-R_0)/a}}{1+ce^{-(r-R_0)/a}}]^2 \right\}. \quad (1)$$

Here, it has four parameters namely H_0, R_0, c and a. With the values of the radial position of the barrier obtained by using global formula R_0=R_B=$r_0(A_1^{1/3} + A_2^{1/3})$+2.72 fm with r_0=1.0 fm, the height of the barrier given by H_0=V_B=$\frac{Z_1Z_2e^2}{R_B}(1-\frac{a_g}{R_B})$ MeV with a_g=0.63 fm along with the values of diffuseness a$\approx$1.9589 fm and constant c=0.6, the effective potential V_{eff}(r) (1) is shown in Fig. 1 as a solid curve. The figure clearly shows that this potential matches almost exactly with the Coulomb+nuclear potential (shown in the same figure by dotted curve) of a typical α+daughter nucleus system with mass number A_1=4 and proton number Z_1=2 of the projectile (α) and mass number A_2=213 and proton number Z_2=87 for the daughter nucleus ^{213}Fr obtained by double folding model.

The s-wave radial Schrödinger equation with the above potential (1) is solved exactly to give the solution as

$$u(r) = A\ z^{ir}(1-z)^{-ir-is}F(p',m',n',\tfrac{1}{1-z}) + B\ z^{-ir}(1-z)^{ir+is}F(p,m,n,\tfrac{1}{1-z}),$$

where z=$-$c $e^{-(r-R_0)/a}$ and F(p,m,n$\frac{1}{1-z}$) is the hyper-geometric function. The other terms are

$p=\frac{1}{2}-ir-it-is$, $m=\frac{1}{2}-ir+it-is$, $n=1-2is$

$p'=\frac{1}{2}+ir-it+is$, $m'=\frac{1}{2}+ir-it+is$, $n'=1+2is$,

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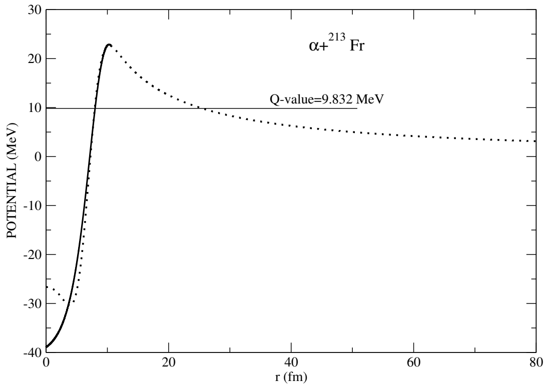

FIG. 1: Plot of Coulomb+nuclear potential as a function of distance for the α+^{213}Fr system. The solid curve represnts the results given by expression (1) with parameters stated in the text. The dotted curve stands for the double folded potential.

r=ka, t=$[q^2(b+1)^2 - \frac{1}{4}]^{1/2}$, s=$[q^2(b^2-1)+r^2]^{1/2}$, k=$\sqrt{\frac{2\mu}{\hbar^2}E}$, $q^2 = \frac{2\mu}{\hbar^2}H_0a^2$, b=$\frac{1}{c}$, reduced mass μ=$\frac{A_1A_2}{A_1+A_2}m_n$ with m_n as mass of nucleon and E stands for the incident energy.

Using the boundary condition u(r=0)=0, we get $z_0 = -c\ e^{R_0/a}$ and

$$C_p = -\frac{B}{A} = z_0^{2ir}(1-z_0)^{-2ir-2is}\frac{F(p',m',n',\frac{1}{1+ce^{R_0/a}})}{F(p,m,n,\frac{1}{1+ce^{R_0/a}})}.$$

As $ce^{R_0/a} >> 1$, $1/(1+ce^{R_0/a}) \simeq 0$, F(p,m,n,0)=1, we get $C_p \simeq e^{-2\pi r}c^{-2is}e^{-2iR_0s/a}$.

In the region $r > R_0 = R_B$ where the potential is pure Coulombic, the Coulomb wave functions (regular F_0 and irregular G_0) for s-wave and their derivatives (F_0' and G_0') are considered. By requirement of continuity at r=R_0, the wave functions and their derivatives are matched at r=R_0 to obtain the S-matrix denoted by S(k) as

$$S(k) = \frac{2ikF_0'-2iF_0f(R_0)+f(R_0)[G_0+iF_0]-k[G_0'+iF_0']}{f(R_0)[G_0+iF_0]-k[G_0'+iF_0']}, \quad (2)$$

where $f(R_0)$=$\frac{du/dr}{u}\ |_{r=R_0}$.

The diffuseness parameter 'a' of the potential (1) is varied around a value 2 to locate the resonance pole k=$k_r - ik_i$ of S(k) at the required Q-value. From this pole, we derive the decay half-life $T_{1/2} = \frac{0.693\mu}{2\hbar k_r k_i}$.

The effective potential (1) with parameters r_0 = 1.0 fm, a_g = 0.63 fm, c=0.6 and a=1.9859 fm having good match with the folded potential of the α+^{213}Fr system is found to generate the resonance at the required energy Q_α=9.832 MeV giving decay half-life $T_{1/2}^{(calt)}$=6.2x10^{-8}s. This result of calculated half-life is in good agreement with the experimental value $T_{1/2}^{(expt)}$=6.9x10^{-8}s having 0.7 uncertainty. Similar agreement is obtained in other cases which are shown in Table 1. We conclude that the half-life calculations for α-decay from the resonance poles of S-matrix of a solvable potential yield good account of the experiment.

TABLE I: Comparison between the experimental [5] α-decay half-lives, $T_{1/2}^{(expt)}$, and calculated half-lives, $T_{1/2}^{(calt)}$, of different α-emitters.

Parent	$E_\alpha^{(expt)}$(MeV)	$T_{1/2}^{(expt)}$(s)	$T_{1/2}^{(calt)}$(s)
^{216}U	8.34	3.80x10^{-3}	4.81x10^{-3}
^{212}Th	7.83	173x10^{-3}	42x10^{-3}
^{208}Fr	6.61	51	21
^{215}Pa	8.08	6.0x10^{-3}	13.7x10^{-3}
^{211}Ac	7.481	78x10^{-3}	70x10^{-3}
^{207}Fr	6.767	1.7	5.5
^{229}Am	7.99	0.9	0.56
^{225}Np	8.63	3.3x10^{-3}	1.2x10^{-3}
^{213}Fr	6.775	16	4.1
^{233}Bk	7.77	21	16

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