Intensity of gravitational waves emitted by pulsar neutron stars due to r-mode oscillation

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Introduction

The recent gravitational wave (GW) signal detected by the LIGO and Virgo collaborations [1]has been confirmed to be emitted from the merger of two neutron stars (NSs). Another continuous source of GW in NSs, theoretically proposed, is due to the r-mode oscillation in pulsar NSs. The r-modes are the neutral circular current on the surface of non-rotating NSs and in the case of pulsars the Coriolis force provides the real dynamics. It has been shown that for the l=m=2 r-modethe surface current persists. Anderson [2] and Friedman & Morsink [3] showed that these r-modes are the prograde in the inertial frame, whereas it is retrograde in its own reference frame that results into emission of GW by the Chandrasekhar – Friedman –Schutz mechanism. The instability resulting due to continuous emission of GW on account of the r-mode is counteracted by the viscous effects in the NS and the oscillation takes place subject to the condition $e^{i\omega t-t/\tau}$, ω being the real part of the r-mode given by $\omega=\frac{(l-1)(l+2)}{(l+1)}\Omega$, where Ω is the angular velocity of the pulsar. The stability of the r-mode oscillation in pulsar NSs is subject to the condition, $\frac{1}{\tau} > 0$, where, $\frac{1}{\tau} = -\frac{1}{\tau_G} + \frac{1}{\tau_{vis}}$, with τ_G being the gravitational time-scale and $\frac{1}{\tau_{vis}}$ is the sum of the reciprocal of all the different viscous time-scales. The viscous effects are the bulk and the shear viscosities in the fluid core and the viscous effect coming from the crust-core boundary layer, $\frac{1}{\tau_{vis}} = \frac{1}{\tau_{BV}} + \frac{1}{\tau_{SV}} + \frac{1}{\tau_{SE}}$, where τ_{BV} and τ_{SV} are bulk and shear time-scales of the core whereas τ_{SE} is the shear viscous time-scale of the crust-core boundary layer. The standard analytical expressions for τ_G, τ_{BV}, τ_{SV} and τ_{SE}, cited in ref [4], are functions of the r-mode angular frequency ω and the temperature *T*. The instability boundary, which is the plot of the critical frequency as a function of *T*, is obtained from the condition $\frac{1}{\tau} = 0$. This instability boundary has been studied in ref.[4] for several equations of state (EOS) constructed from the finite range simple effective interaction (SEI) for different values of the slope parameter $L(\rho_0)$ and nuclear matter incompressibilities $K(\rho_0)$ (See ref.[4] for details).

The temperature $T > 10^{11}$ K in newly born NSs and the star enters the region of instability as it cools. In old cold NSs in a binary system, the star can enter the region of instability as it accretes mass from its companion. The r-mode amplitude α of a pulsar NS in the region of instability goes on increasing till a saturationis reached. At this point it emits a massive GW radiating energy and angular momentum and spins down to the region of stability. The spin-down rate can be calculated from the observed luminosity of the NS. The intensity of the emitted GW can be estimated from the amplitude of the GW strain tensor h_0.

Formalism

The GW amplitude, i.e. strain tensor amplitude, h_0, is given by [5, 6]

$$h_0 = \sqrt{\frac{8\pi}{5}}\frac{G}{c^5}\frac{1}{r_0}\alpha\omega^3 M R^3 \tilde{J} \quad \text{.....(1)}$$

where, G is the gravitational constant, c is the velocity of light, r_0 is the distance of the source, M and R are the mass and the radius of the NS and $\tilde{J}=\frac{1}{MR^4}\int_0^R \rho(r) r^6 dr$, ρ being the mass density. Under the consideration of standard neutrino cooling, the amplitude α of the r-mode is given by

$$\alpha = 7.9494\times 10^{-17}\left[\frac{-\tau_G}{\tilde{J}}\right]^{1/2}\frac{\sigma^{1/2}T_{eff}^2}{\Omega}\left[\frac{M_0}{M}\right]^{1/2} \quad \text{.....(2)}$$

where $T_{\rm eff}$ is the surface temperature. The computation of h_0 in equation (1) can be done provided the radial dependence of the mass density in the NS is known.

Results and discussion

The GW strain amplitude tensor h_0 is calculated for 1.4 $M_\odot$ NSs from equation (1) using the EOS of SEI corresponding to γ=1/2 (see ref [4] for details). The results for nine NSs which are predicted in the instability region as per ref [4] are given in Table 1. The maximum magnitude of h_0 is of the order 10^{-28} which is much below the detectability efficiency of the advanced LIGO. This result is inconformity with the values found using the microscopic APR EOS [7]. Further, in order to examine the influence of the angular velocity of the NS, Ω, surface temperature, $T_{\rm eff}$, and the distance of the source, r_0, on the amplitude of the strain tensor, h_0, we have calculated h_0 for two representative values of the spin frequency, ν= Ω/2π= 50 Hz and 650 Hz for 1.4 $M_\odot$NSs. For each case the study has been made for two values of the surface temperature, $T_{\rm eff}$ =150 eVand 50 eV. The results are shown Figure 1 as a function of distance r_0. It can be seen that the NS having lower Ω for a given $T_{\rm eff}$ will emit GW of higher intensity. Also the intensity of the GW will increase with rise in the surface temperature $T_{\rm eff}$.

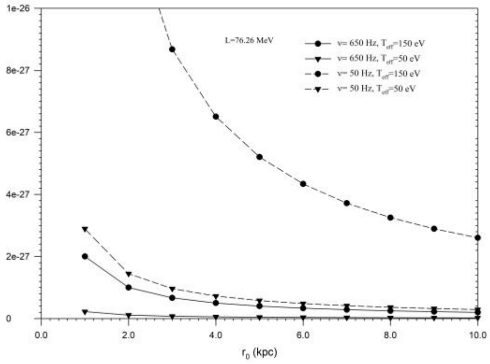
Conclusion

The order of magnitude of h_0 of the GW emitted by a NS at saturation in the region of instability is 10^{-28} which is below the detectability limit of advanced LIGO. Further it is found that for NSs having same masses and surface temperature, the star rotating with lower angular velocity and having higher surface temperature shall emit GW of higher intensity.

Table1: The amplitude of strain tensor h_0 for the EOS γ=1/2 (L=76.26 MeV) for 1.4 $M_\odot$ NS (ν, r_0 and $T_{\rm eff}$ are from references cited in [4]).

Source	ν	h_0
	(Hz)	(1.4 $M_\odot$)
4U1608-522	620	$6.57^{-0.58}_{+0.72}$ x10^{-29}
IGR J00291+5934	599	$9.74^{-1.63}_{+2.46}$ x10^{-29}
MXB 1659-29	556	$2.74^{-0.32}_{+0.41}$ x10^{-29}
Aql X-1	550	$2.04^{-0.46}_{+0.86}$ x10^{-28}
KS 1731-260	524	$7.51^{-0.91}_{+1.21}$ x10^{-29}
XTE J1751-305	435	$7.45^{-1.86}_{+3.73}$ x10^{-29}
SAX J1808-3658	401	$3.71^{-0.10}_{+0.10}$ x10^{-29}
XTE J1814-338	314	$1.31^{-3.96}_{+1.00}$ x10^{-28}
NGC 6440	205	$2.51^{-0.11}_{+0.13}$ x10^{-28}

Figure1: h_0 shown as a function of distance r_0 for 1.4$M_\odot$ NS for two representative values of the frequency, ν = 650 Hz and 50 Hz, and for surface temperatures, $T_{\rm eff}$ = 150 eV and 50 eV, using the EOS γ =1/2 of SEI.



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