Influence of transition density calculated by thermodynamical method and dynamical method on crustal fraction of moment of inertia of neutron star using Rsigma and Gsigma Skyrme interaction

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Introduction

The Crust-Core transition density (ρ_t) in Neutron Stars (NSs) plays a very important role in different phenomena exhibited by NSs. It is popularly predicted by Thermodynamical Method (TDM) [1]. In this method the gradient and coulomb contributions are not taken into account. The more rigorous way to determine the transition density by including the coulomb and density gradient terms is known as Dynamical Method (DM) [2]. In the present work our objective is to examine the influence of the Crust-Core transition density (ρ_t) calculated by TDM and DM on one of the properties of the NSs, i.e., Crustal fraction of moment of inertia $(\frac{\Delta I}{I})$, where the NS crust plays important role. This study has been carried out by using Rsigma and Gsigma Skyrme Interaction.

Formalism

The transition density (ρ_t) is calculated from the onset of instability of the uniform liquid against small amplitude density fluctuation due to clusterization. This is examined by analyzing the thermodynamical stability conditions in the β-equilibrated dense n+p+e+μ matter which is given by [1],

$$V_{thermal} = 2\rho\left(\frac{\partial e(\rho,Y_p)}{\partial \rho}\right) + \rho^2\left(\frac{\partial^2 e(\rho,Y_p)}{\partial \rho^2}\right) - \frac{\left(\rho\frac{\partial^2 e(\rho,Y_p)}{\partial\rho\partial Y_p}\right)^2}{\left(\frac{\partial^2 e(\rho,Y_p)}{\partial {Y_p}^2}\right)} > 0 \quad \ldots(1)$$

Where, $e(\rho,Y_p)$ is the energy per baryon, ρ is the total nucleon density of nucleon and Y_p is the proton fraction.

In dynamical approach, the stability condition for a homogeneous NS matter against small density fluctuations can be approximated as [2],

$$V_{dyn}(k) = V_0 + \beta k^2 + \frac{4\pi e^2}{k^2+k_{TF}^2} > 0, \quad \ldots (2)$$

Where k is the wave vector of the spatially periodic density perturbations and

$$V_0 = \frac{\partial\mu_p}{\partial\rho_p} - \frac{(\partial\mu_n/\partial\rho_p)^2}{\partial\mu_n/\partial\rho_n}, \quad k_{TF}^2 = \frac{4\pi e^2}{\partial\mu_e/\partial\rho_e},$$
$$\beta = D_{pp} + 2D_{np}\zeta + D_{nn}\zeta^2, \; \zeta = -\frac{\partial\mu_n/\partial\rho_p}{\partial\mu_n/\partial\rho_n} \quad \ldots\ldots (3)$$

The three terms in Eq. (2) represents the contributions from the bulk nuclear matter, the density gradient terms and the coulomb interaction respectively. The coefficients D_{ij} *(i,j = n,p)* are expressed in terms of the Skyrme interaction parameters t_1, x_1, t_2 and x_2 as [3],

$$D_{nn} = D_{pp} = \frac{3}{16}[t_1(1-x_1) - t_2(1x_2)] \quad \ldots (4)$$
$$D_{np} = D_{pn} = \frac{1}{16}[3t_1(2+x_1) - t_2(2+x_2)] \quad ..(5)$$

The dynamical approach reduces to thermodynamical approach in the long wavelength limit when the density gradient terms and coulomb interaction are neglected, which leads to the stability condition

$$V_{thermal} = \frac{\partial\mu_p}{\partial\rho_p} - \frac{(\partial\mu_n/\partial\rho_p)^2}{\partial\mu_n/\partial\rho_n} > 0 \quad \ldots\ldots (6)$$

The crustal fraction of the moment of inertia (ΔI/I) contains the mass M and radius R of the NS and is given by the following approximate expression [4]

$$\frac{\Delta I}{I} \approx \frac{28\pi P(\rho_t)R^3}{3Mc^2}\left(\frac{1-1.67\xi-0.6\xi}{\xi}\right) \times \left(1 + \frac{2P(\rho_t)}{\rho_t mc^2}\frac{(1+7\xi)(1-2\xi)}{\xi^2}\right)^{-1} \quad \ldots (7)$$

Where $\xi = \frac{GM}{RC^2}$, G is the gravitational constant, R is the radius of NS and c is the velocity of light.

The Skyrme interaction used in the present study is given by [5]

$$V(\rho_n, \rho_p) = \frac{t_0}{4}\left[(2+x_0)\rho^2 - (1+2x_0)(\rho_n^2 + \rho_p^2)\right] + \frac{t_3}{24}\rho^\gamma\left[(2+x_3)\rho^2 - (1+2x_3)(\rho_n^2 + \rho_p^2)\right] + \frac{1}{8}\rho^\gamma\left[t_2(1+2x_2) - t_1(1+2x_1)\right](\tau_n\rho_n + \tau_p\rho_p) + \frac{1}{8}\rho^\gamma\left[t_1(2+x_1) + t_2(2+x_2)\right]\tau\rho \quad \ldots (8)$$

Where, $\rho_{n(p)}$ is the neutron (proton) density, $\tau_i = \frac{3}{5}k_i^2\rho_i$, $\tau = \tau_p + \tau_n$, and i= n,p. The skyrme interaction altogether contain nine parameters, namely, γ, t_0, x_0, t_1, x_1, t_2, x_2, t_3, x_3. The Gsigma and Rsigma Skyrme interactions have been used in the present work.

Results and Discussion

The calculation of the crustal fraction of the moment of inertia requires the transition density and pressure at the transition density apart from the bulk properties of the NS. The transition density and pressure is calculated in both the thermodynamical spinoidal formulation and the Dynamical method. These calculations are made using the Gsigma and Rsigma Skyrme interactions. The neutron star core-crust transition density and corresponding transition pressure are given in Table-1. From the table it can be seen that the transition density has a lesser value in case of DM than that of TDM. The crustal fraction of the moment of inertia as a function of central density ρ_c is calculated for Gsigma and Rsigma skyrme model. The result is listed in Table-2. It can be analysed from Table-2 that crustal fraction of moment of inertia decreases upto nearly 40% in the case of dynamical method.

Table 1. The values of Transition Density (ρ_t) in the unit of fm^{-3} and the corresponding Pressure (P_t) in the unit of MeV fm^{-3} using TDM and DM with Skyrme interaction.

Skyrme Interacti-on	TDM		DM	
	ρ_t	P_t	ρ_t	P_t
Gsigma	0.0638	0.281	0.0531	0.155
Rsigma	0.0672	0.307	0.0329	0.184

Table 2. The Crustal fraction of moment of inertia as a function of central density ρ_c of the star using TDM and DM with Gsigma and Rsigma Skyrme interaction.

ρ_c fm^{-3}	Crustal Fraction of moment of inertia ΔI/I			
	Gsigma		Rsigma	
	TDM	DM	TDM	DM
2.0	.0034	.0020	.0036	.0021
1.9	.0035	.0020	.0038	.0022
1.8	.0037	.0021	.0039	.0023
1.7	.0038	.0022	.0041	.0024
1.6	.0040	.0023	.0043	.0025
1.5	.0043	.0024	.0045	.0027
1.4	.0046	.0026	.0049	.0029
1.3	.0050	.0029	.0053	.0031
1.2	.0055	.0032	.0059	.0034
1.0	.0071	.0036	.0066	.0039
0.9	.0085	.0049	.0090	.0052
0.8	.0104	.0060	.0111	.0064
0.7	.0132	.0078	.0142	.0082
0.6	.0178	.0106	.0191	.0109
0.5	.0257	.0156	.0276	.0155
0.4	.0400	.0252	.0432	.0238
0.3	.0684	.0459	.0742	.0396
0.2	.1295	.0963	.1405	.0715

References

[1] S. Kubis, Phys. Rev. C 76,025801 (2007)

[2] G.Baym, C. Pethik and P. Sutherland, Astrophys. J. 170, 299 (1971).

[3] E. Chabanat, P. Bonche, P. Hensel, J. Meyer, and R. Schaeffer, Nucl. Phys. A 627,710 (1997)

[4] J. Lattimer, M. Prakash, Astrophysics. J. 550, 426 (2001).

[5] M. Dutra, O. Lourenco, J.S.S. Martins, A. Delfino, J.R. Stone, P.D. Stevenson, Phys. Rev. C 85, 035201 (2012).