Deep sub-barrier fusion of p+^{6,7}Li, p+D and S-function

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Introduction

The cross sections for the deep sub-barrier fusion reactions of light nuclei play important role in big-bang and stellar nucleosynthesis. This work calculates p+^{6,7}Li, p+D fusion cross sections within the theoretical framework of the selective resonant tunneling model. In this model, assumption of a complex square-well nuclear potential is invoked to describe the absorption inside the nuclear well. The theoretical estimates for these cross sections agree well with the experimentally measured values. The features of the astrophysical S-factor are derived in terms of this model.

Theoretical formalism

In the fusion of light nuclei, the relative motion can be described in terms of the reduced radial wave function $\zeta(r)$ given by $\Phi(r,t) = \frac{1}{\sqrt{4\pi}r}\zeta(r)\exp(-i\frac{E}{\hbar}t)$ where $\Phi(r,t)$ represents the solution of the general Schrödinger equation for the system. The reaction cross section in terms of the phase shift δ_0 due to the nuclear potential (in low energy limit only s-wave contributes) is given by $\sigma = \frac{\pi}{k^2}[1-|\eta|^2]$ where $\eta = e^{2i\delta_0}$ and k is the wave number for relative motion. Nuclear potential being complex, the corresponding phase shift δ_0 is complex and given by [1]

$$cot(\delta_0) = W_r + iW_i. \quad (1)$$

Consequently, the cross section is given by

$$\sigma = \frac{\pi}{k^2}\left\{-\frac{4W_i}{(1-W_i)^2+W_r^2}\right\} \quad (2)$$
$$= \left(\frac{\pi}{k^2}\right)\left(\frac{1}{\chi^2}\right)\left\{-\frac{4\omega_i}{\omega_r^2+(\omega_i-\frac{1}{\chi^2})^2}\right\}$$

where $\chi^2 = \frac{\exp(\frac{2\pi}{ka_c})-1}{2\pi}$ is the Gamow penetration factor, $k=\sqrt{\frac{2\mu E}{\hbar^2}}$ is wave number outside nuclear well and $a_c = \frac{\hbar^2}{Z_1Z_2\mu e^2} = \frac{1}{k\zeta}$ is Coulomb unit of length. The dimensionless term

$$S_f(E) = -\frac{4\omega_i}{\omega_r^2+(\omega_i-\frac{1}{\chi^2})^2} \quad (3)$$

is called the astrophysical S-function and is related to the astrophysical S-factor $S(E)$ by $S(E) = \frac{\pi\hbar^2\exp(2\pi\zeta)}{2\mu\chi^2}S_f(E)$, where $\omega = \omega_r + i\omega_i = (W_r + iW_i)/\chi^2$. The wave function inside the nuclear well is determined by two parameters, the real V_r and the imaginary part V_i of the nuclear potential. The Coulomb wave function outside the nuclear well is determined by two other parameters: the real, imaginary part of the complex phase shifts δ_{0r}, δ_{0i}. A pair of convenient parameters W_r, W_i are introduced to make a linkage between the cross section and the nuclear well for describing the resonance and the selectivity in damping. The boundary condition for the wave function can be expressed by its logarithmic derivative which for the square well is given by

$$R\frac{[\sin(Kr)]'}{\sin(Kr)}|_{r=R} = KR\cot(KR) \quad (4)$$

and in the Coulomb field, it is given by

$$\frac{R}{a_c}\left\{\frac{1}{\chi^2}\cot(\delta_0)+2\left[\ln\left(\frac{2R}{a_c}\right)+2A+y(ka_c)\right]\right\} \quad (5)$$

so that

$$\omega_i = W_i/\chi^2 = Im\left[\frac{a_c}{R}(KR)\cot(KR)\right] \quad (6)$$
$$= \frac{a_c}{R}\left\{\frac{\gamma_i\sin(2\gamma_r)-\gamma_r\sinh(2\gamma_i)}{2[\sin^2(\gamma_r)+\sinh^2(\gamma_i)]}\right\}$$
$$\omega_r = W_r/\chi^2 = \frac{a_c}{R}\left\{\frac{\gamma_r\sin(2\gamma_r)+\gamma_i\sinh(2\gamma_i)}{2[\sin^2(\gamma_r)+\sinh^2(\gamma_i)]}\right\} - 2H \quad (7)$$

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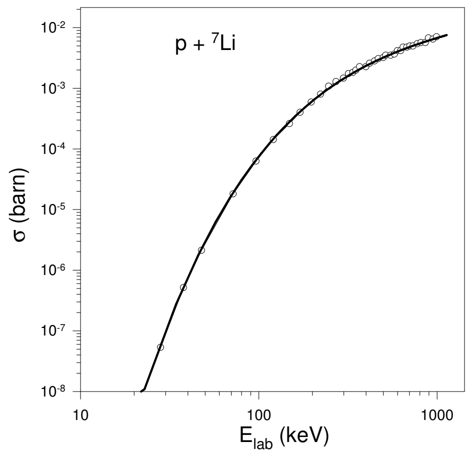

FIG. 1: Plot of cross-section as a function of lab energy for p+^{7}Li fusion reaction.

where $K^2=\frac{2\mu}{\hbar^2}[E-(V_r+iV_i)]$, $K_i=\frac{\mu}{K_r\hbar^2}(-V_i)$, $\gamma=(\gamma_r+i\gamma_i)\equiv(K_rR+iK_iR)$, radius of nuclear well $R=r_0(A_1^{1/3}+A_2^{1/3})$, r_0 is the radius parameter, A_1, A_2 are the mass numbers of the reacting nuclei, $H=\left[\ln\left(\frac{2R}{a_c}\right)+2A+y(ka_c)\right]$, $y(x)=\frac{1}{x^2}\sum_{j=1}^{\infty}\frac{1}{j(j^2+x^{-2})}$ and A=0.577 is the Euler's constant, $y(ka_c)$ is related to the logarithmic derivative of Γ function given as y(x).

Calculations and Results

The fusion cross sections and the astrophysical S-function are calculated using Eq.(2) and Eq.(3), respectively. Plot of cross-section as a function of lab energy for p+^{7}Li fusion reaction (continuous line) is shown in Fig.1 and compared with experimental data [2] (hollow circles). Plots of S-function as a function of lab energy for p+^{6}Li, p+^{7}Li and p+D fusion reactions are shown in Fig.2. The magnitudes of V_r, V_i and r_0 for these three cases are, respectively, −55.0 MeV, −0.0235 keV, 1.177 fm, −44.25 MeV, −7.5 MeV, 1.180 fm and −85.0 MeV, −395.0 keV, 1.330 fm. It is interesting to note that the magnitude of V_i for fusion of p+^{6}Li is about twenty times larger compared to that of p+^{7}Li. The reason may be attributed to extremely low lifetime of ^{8}Be inhibiting its formation. The reason for extremely small V_i in case of p+D system lies in the fact that the binding energy for deuteron is very small and therefore it easily disintegrates into p+n causing very small p+D fusion cross section and correspondingly very small V_i.

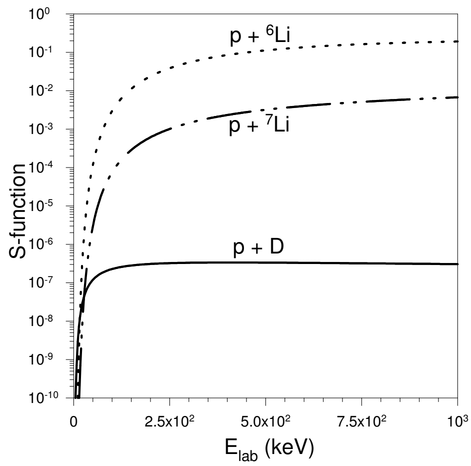

FIG. 2: Plots of S-function as a function of lab energy for p+^{6}Li, p+^{7}Li and p+D fusion reactions.

Summary and Conclusion

In the deep sub-barrier fusion of light nuclei, the nuclear resonance selects not only the frequency or the energy level but also the damping that causes nuclear reaction. When the resonance occurs, the selectivity becomes very sharp. In such selective resonant tunneling processes the neutron-emission reaction is suppressed. The process of fusion of light nuclei at very low energies can recall the phase factor of the wave function describing the system. The imaginary part in the square well potential describes the formation of compound nucleus formed by the fusion process, but there are not enough collisions to justify the assumptions for compound nucleus model in case of light nuclei. The p+^{6}Li, p+^{7}Li and p+D fusion reactions are of astrophysical importance. It is interesting to notice that while the real part of the potential is mainly derived from the resonance energy, the imaginary part of the potential is determined by the Gamow factor at resonance energy. The good agreement between the experimental data and the quantum-mechanical calculation suggests strongly of selective resonant tunneling.

References

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