Charmonia within an instanton potential

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Introduction

Heavy quark physics evolved to a new phase. Novel findings of heavy hadrons renewed the interest in the heavy quark spectra. Precise measurement of the mass spectra provides the subtle test for any phenomenological model about quarkonia, based on Quantum chromodynamics. Charmonium are bound states of charm and anti-charm quark and several quarkonium states have been observed after the discovery of J/ψ at BNL and SLAC [1]. The first singlet ground state of charmonium η_c observed in the decays of J/ψ and ψ′ [1]. The discoveries of the other states reported by PDG like h_c(1P), h_c(2P), χ_c (1P), χ_c (2P) , exotic states like X(3872), X(3915), Y(4560), Z(3930) at Belle, BaBar, LHC, BESIII, CLEO are the thrust area of interest for experimentalists as well as theorists. To study the underlying field theory for such systems one has to consider QCD. But due to the difficulty in finding analytically approximated solutions, alternative approaches like numeric Lattice QCD, Potential quark models, unquenched quark models, Bethe-Salpeter are followed.

Quark confinement is the most important non-perturbative part which can be obtained phenomenologically from the Wilson loop for heavy quark potential, which rises most linearly at the larger distances. This linearly rising potential is crucial in lattice QCD. Another non-perturbative effect on heavy quarks potential is from instantons. Instatons are essential topological objects in description of lattice QCD.

Theoretical Framework

The framework of Instanton liquid model have two important parameters [2]. One is the average size of the instanton (ρ) and other is average distance between instantones (R). Lattice simulations of the instanton vacuum gives the numeric value for ρ and R as 0.36 fm and 0.89 fm, respectively which we have used in present work. But such values one cannot consider as exact. There are other sets of values emerging from the different approaches. And it will be of interest to check the dependence of the heavy quark potential on different sets of parameters. The central part of the potential is derived by averaging the Wilson loop in the instanton ensemble [2]. To derive the spin dependent aspects, matter part of QCD Lagrangian for heavy quark is expanded with respect to inverse of the heavy quark mass, similarly as one generally do in heavy quark effective theory [3]. Finally it can be shown that heavy quark propagator can be written as an integral equation [4].

When the distance between quark-anti quark is smaller than the size of instantons, one can expand the dimensionless integral I(x) in terms of x [4] and this in turn gives the central part of the potential having polynomial form as [4]

$$V_C = \frac{4\pi\rho^3}{R^4 N_C}\left(1.345\frac{r^2}{\rho^2} - 0.501\frac{r^4}{\rho^4}\right) \tag{1}$$

As distance between quark and antiquark becomes larger than the size of instanton, the form of the potential comes out to be [4]

$$V_C(r) = 2\Delta M_Q - \frac{g_{NP}}{r} \tag{2}$$

$$g_{NP} = \frac{2\pi^3\rho^4}{N_C R^4} \tag{3}$$

In the above equation the second term acts like a Coulomb potential, so this sequentially understood as a non perturbative contribution to the perturbative one gluon exchange potential by instanton medium at larger r. Here, g_{NP} can be treated as non perturbative correction to the strong coupling constant α_s. Moreover, when r tends to infinity the potential is saturated at the value of $2\Delta M_Q$. ΔM_Q is the correction to the heavy quark mass from instanton vacuum and that used in the present work is 135.72 MeV.

To solve the bound states for quarkonium mass spectra, we need to solve the Schrödinger equation having instanton vacuum induced potential. Rayleigh - Ritz variational method is then employed. The trial wave function employed here is,

$$\varphi_n^L(r) = \left(\frac{2^{2L+\frac{7}{2}}\mu^{-2L-3}}{\sqrt{\pi}\,(2L+1)!!}\right)^{\frac{1}{2}} r^L e^{-\left(\frac{r}{\mu}\right)^2} \quad (4)$$

Where, μ is the variational parameter. The ground state energy is acquired by minimizing the expectation value of $\mathcal{H}$.

Results and Discussions

Table 1 comprises preliminary results for the nS charmonium states. We have compared present results with the experimental, NRQM [5], Lattice QCD [6] and other potential model [7-8].

The parameters in our model are fixed by referring the mass spectra. Different form of the potential can also give the identical results so for a particular model, predictions of other observables like annihilation and radiative decay widths are necessary to better understand the nature of inter quark interactions and decay processes. It will be important to study the mass spectra of the quarkonia by solving the Schrödinger equation explicitly, with heavy quark potential from instaton vacuum and confining potential together. The corresponding analysis are under way. Instanton vacuum plays an important role in realization of chiral symmetry and its spontaneous breaking in QCD. Starting with the operators corresponding to Wilson line, we can consider next and next to next leading orders in expansion with respect to the small packing parameter in the instanton vacuum and by these attempts we could see the better results for the quarkonium properties.

Table 1: Mass spectra for nS Charmonium states

State	Present MeV	Exp. MeV	[5] MeV	[6] MeV	[7] MeV	[8] MeV
1^1S_0	3027	2984	3088	3010	2980	2977
1^3S_1	3082	3097	3168	3085	3097	3096
2^1S_0	3717	3639	3669	3770	3631	3630
2^3S_1	3544	3686	3707	3739	3687	3684
3^1S_0	3915	3940	4067	-	3992	3990
3^3S_1	4206	4040	4094	-	4030	4022

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