Doubly heavy baryon in the relativistic quark model

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Introduction

Quarks inside the baryon forms pairwise cluster and leading to a simple two-body structure. This diquark approach is used to understand the structure and decay properties of baryons and exotic hadrons. Here, we study the mass spectra of the doubly heavy spin - $\frac{1}{2}$ and spin - $\frac{3}{2}$ baryons containing two heavy quarks (Ξ_{cc}, Ω_{cc}, Ξ_{bb} and Ω_{bb}) in the bottom and charm sector within the diquark-quark picture, in a relativistic formalism.

Theoretical Methodology

The interaction between the two quarks forming a diquark is assumed to be in the form of linear potential. The form of the model potential is thus expressed as [1],

$$V(r) = \frac{1}{2}(1+\gamma_0)(\lambda r + V_0) \qquad (1)$$

Here, λ is the strength of the confinement part of the potential. V_0 is a constant negative potential depth.

The wave functions $\psi(\vec{r})$ satisfies the Dirac equation given by,

$$[\gamma^0 E_q - \vec{\gamma}.\vec{P} - m_q - V(r)]\psi_q(\vec{r}) = 0. \qquad (2)$$

To get the binding energy of the quarks, we have solved the two component Dirac equation [1–4]. As we are interested to have the energy of the quarks (positive energy) only, the positive energy solution can be expressed as,

$$\psi^{(+)}_{nlj}(\vec{r}) = N_{nlj}\begin{pmatrix} ig(r)/r \\ (\sigma.\hat{r})f(r)/r \end{pmatrix}\mathcal{Y}_{ljm}(\hat{r}) \qquad (3)$$

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TABLE I: Masses of cc and bb diquark (in MeV).

Diquark content	state	Mass	[5]	[6]
{cc}	1^3S_1	2933	3294	3226
{cc}	2^3S_1	3287	3603	3535
{bb}	1^3S_1	9479	9823	9778
{bb}	2^3S_1	9886	10088	10015

TABLE II: Predicted mass spectra of Ξ_{cc} and Ω_{cc} (in MeV).

State	Ξ_{cc}			Ω_{cc}		
$N_dL_dn_ql_q$ (J^P)	our	[5]	[6]	our	[5]	[6]
(1S 1s)$(\frac{1}{2})^+$	3635	3606	3620	3733	3715	3778
(1S 1s)$(\frac{3}{2})^+$	3854	3675	3727	3912	3772	3872
(1S 2s)$(\frac{1}{2})^+$	4118	4172	· · ·	4162	4270	· · ·
(1S 2s)$(\frac{3}{2})^+$	4276	4193	· · ·	4294	4288	· · ·
(2S 1s)$(\frac{1}{2})^+$	3989	4004	3910	4079	4118	4075
(2S 1s)$(\frac{3}{2})^+$	4196	4036	4027	4259	4142	4174
(2S 2s)$(\frac{1}{2})^+$	4461	· · ·	· · ·	4503	· · ·	· · ·
(2S 2s)$(\frac{3}{2})^+$	4611	· · ·	· · ·	4634	· · ·	· · ·

with the energy eigen value as [1],

$$\epsilon = (E_q - m_q - V_0)(m_q + E_q)^{\frac{1}{3}}\lambda^{\frac{-2}{3}} \qquad (4)$$

Our aim is to interpret the three-quark bound state in diquark-quark like structure, the quark-quark interaction in a diquark state is half of the potential between a quark Q and an antiquark $\bar{Q}$. Therefore we adopt the relation $V_{QQ} = \frac{1}{2}V_{Q\bar{Q}}$ to derive the diquark masses [5]. The diquark masses can then be expressed as,

$$M_d = 2(E_Q + m_Q) - E_{d(c.m)} \qquad (5)$$

and further, the procedure is repeated for the quark-diquark interaction leading to the mass of baryonic states as,

$$M_{QQq} = (M_d+E_d)+(m_q+E_q)-E_{dq(c.m)} \qquad (6)$$

Here, 'd' represent diquark and E_{dq} represents the energy eigenvalue of the diquark - quark system, respectively. The center of mass correction to the diquark state ($E_d(c.m)$) and that of the 'dq' system $E_{dq(c.m)}$ are absorbed in the potential parameter V_0 of eqn.(1). The j-j coupling interaction is given by,

$$\langle V_{QQq}^{j_d j_q} \rangle = \frac{\sigma \langle j_d j_q JM | \hat{j_d} \cdot \hat{j_q} | j_d j_q JM \rangle}{(E_d + M_d)(E_q + m_q)} \quad (7)$$

Result and discussion

The ground and excited states of doubly heavy baryons are still not clear to us experimentally (except Ξ_{cc}^{+} and Ξ_{cc}^{++}). In present work, we study the mass spectra of Ξ_{cc}, Ω_{cc}, Ξ_{bb} and Ω_{bb} within the diquark-quark picture in a relativized quark model. To obtain the masses of diquark and baryons we have solved Dirac equations with a linear confinement potential. Besides the ground state of doubly heavy baryons, the masses for the low-lying excited doubly charmed and bottom baryons are listed in Table II and III. We have also compared our predicted results for radial excited states with available theoretical results. Very recently R. Aaij et.al (LHCb collaboration) have measured mass of Ξ_{cc}^{++} (ccu) as $3620.6 \pm 1.5(stat.) \pm 0.4(syst.) \pm 0.3(\Xi_c^{+})$ $\frac{MeV}{c^2}$ [7] is in accordance with our predicted mass of 3634 MeV. The masses are computed for

TABLE III: Predicted mass spectra of Ξ_{bb} and Ω_{bb} (in MeV).

State	Ξ_{bb}			Ω_{bb}		
$(N_d L_d n_q l_q)(J^P)$	our	[5]	[6]	our	[5]	[6]
(1S 1s)$(\frac{1}{2})^+$	10302	10138	10202	10349	10230	10359
(1S 1s)$(\frac{3}{2})^+$	10589	10169	10237	10593	10258	10389
(1S 2s)$(\frac{1}{2})^+$	10942	10662	10832	10905	10751	10970
(1S 2s)$(\frac{3}{2})^+$	11139	10675	10860	11082	10763	10992
(2S 1s)$(\frac{1}{2})^+$	10623	10464	$\cdots$	10672	10566	$\cdots$
(2S 1s)$(\frac{3}{2})^+$	10912	10480	$\cdots$	10913	10579	$\cdots$
(2S 2s)$(\frac{1}{2})^+$	11226	$\cdots$	$\cdots$	11224	$\cdots$	$\cdots$
(2S 2s)$(\frac{3}{2})^+$	11456	$\cdots$	$\cdots$	11403	$\cdots$	$\cdots$

both $J^P = (\frac{1}{2})^+$ and $(\frac{3}{2})^+$. The Spin average masses and hyperfine splitting are also predicted by the model in Table IV and V. For the hyperfine splitting we have employed the confined one gluon exchange potential [1–3].

TABLE IV: Spin average masses of Ξ_{cc}, Ω_{cc}, Ξ_{bb} and Ω_{bb} (in MeV).

$(N_d L_d n_q l_q)$	Ξ_{cc}	Ω_{cc}	Ξ_{bb}	Ω_{bb}
(1S 1s)	3781	3852	10489	10511
(1S 2s)	4223	4250	11073	11023
(2S 1s)	4127	4199	10816	10833
(2S 2s)	4561	4590	11379	11340

TABLE V: Hyperfine splitting of Ξ_{cc}, Ω_{cc}, Ξ_{bb} and Ω_{bb} (in MeV).

$(N_d L_d n_q l_q)$	Ξ_{cc}	Ω_{cc}	Ξ_{bb}	Ω_{bb}
(1S 1s)$[(\frac{3}{2}) - (\frac{1}{2})]$	219	179	287	244
(1S 2s)$[(\frac{3}{2}) - (\frac{1}{2})]$	158	132	197	177
(2S 1s)$[(\frac{3}{2}) - (\frac{1}{2})]$	207	180	289	241
(2S 2s)$[(\frac{3}{2}) - (\frac{1}{2})]$	155	131	230	179

Acknowledgments

We acknowledge the financial support from DST-SERB, India (research Project number: SERB/F/8749/2015-16).

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