# Wigner distributions of an electron in simulated QED model

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## Introduction

In quantum electrodynamics (QED), the electron is an elementary particle which has no substructure. But according to Heisenberg uncertainty principle, an electron can fluctuate into virtual electron-photon pair i.e. $e \to e\gamma \to e$. Further, virtual photons can break up into more pairs of virtual electron and positron. Therefore, in quantum theory a bare electron can be treated as it is surrounded by the cloud of virtual electrons, positrons and photons. As a result a bare electron become a dressed electron. The virtual cloud interaction with a probe reveals the structure of the electron [1, 2].
The multi-dimensional phase-space distributions of an particle is provide by Wigner distributions which can provide combined information of the position and momentum distribution. In this work, we study the Wigner distributions of an electron using the simulated QED model of leptons in which the behaviour of wave functions is enhanced at the end points at $x = 0, 1$ [3, 4].

## Wigner distributions

One can defined the Wigner distributions as

$$\rho^{[\Gamma]}(\mathbf{b}_\perp, \mathbf{k}_\perp, x; S) = \int \frac{d^2\Delta_\perp}{4\pi^2} e^{-i\mathbf{\Delta}_\perp . \mathbf{b}_\perp} \times \int \frac{dy^- d^2\boldsymbol{y}_T}{16\pi^3} \frac{e^{ik.y}}{2} \times \langle P, S|\bar{\psi}(0)\psi(y)|P, S\rangle\Big|_{y^+=0},$$

where $\Gamma$ is the Dirac matrix $(\gamma^+, \gamma^+\gamma_5, i\sigma^{j+}\gamma_5)$. The Wigner distributions of the electron are evaluated by considering it as a two particle system of internal electron and photon. The Winger distributions for unpolarized and longitudinal polarized electron by using overlap representation of LFWFs are expressed as

$$\rho^{[\gamma^+]}(\mathbf{b}_\perp, \mathbf{k}_\perp, x; S) = \int \frac{d^2\mathbf{\Delta}_\perp}{4\pi^2} e^{-i\mathbf{\Delta}_\perp . \mathbf{b}_\perp} \times \sum_{\lambda_{1'}, \lambda_1, \lambda_2} \Psi^{*\Lambda'}_{\lambda_{1'}, \lambda_2} \; \chi^\dagger_{\lambda_{1'}} \chi_{\lambda_1} \Psi^{\Lambda}_{\lambda_1, \lambda_2}.$$

The unplorized and longitudinal polarized Wigner distributions are defined as:

$$\rho_{UU}(\mathbf{b}_\perp, \mathbf{k}_\perp, x) = \frac{1}{2}\Big[\rho^{[\gamma^+]}(\mathbf{b}_\perp, \mathbf{k}_\perp, x; e_z) + \rho^{[\gamma^+]}(\mathbf{b}_\perp, \mathbf{k}_\perp, x; -e_z)\Big],$$

$$\rho_{LU}(\mathbf{b}_\perp, \mathbf{k}_\perp, x) = \frac{1}{2}\Big[\rho^{[\gamma^+]}(\mathbf{b}_\perp, \mathbf{k}_\perp, x; e_z) - \rho^{[\gamma^+]}(\mathbf{b}_\perp, \mathbf{k}_\perp, x; -e_z)\Big].$$

We have used the generalized form of QED by assigning a mass $M$ to the external electrons and a different mass $m$ to the internal electron and a mass $\lambda$ to the internal photon. The wave functions for the two-particle Fock state of an electron with up helicity and down helicity are given in [6]. To derive the simulated QED model, light-front wave function $\varphi(x, \mathbf{k}_\perp)$ in Eq. (5) is differentiated with respect to $M^2$ as in [4]. In other words, we take

$$\varphi^{'}(x, \mathbf{k}_\perp) = M^2 \frac{\partial \varphi(x, \mathbf{k}_\perp)}{\partial M^2}. \qquad (1)$$

This improves the convergence of the wave functions at the end points of $x$ as well as the $\mathbf{k}_\perp^2$ behaviour as shown in [5].

(a)

b)

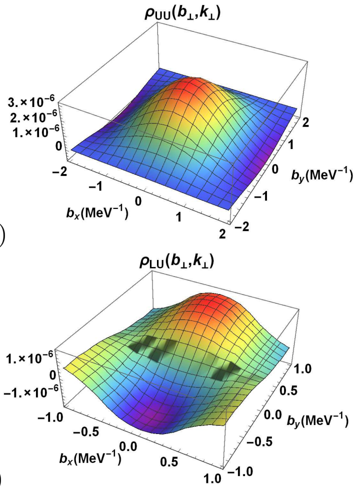


FIG. 1: Wigner distributions $\rho_{UU}(b_\perp, k_\perp)$ and $\rho_{LU}(b_\perp, p_\perp)$ in impact-parameter space at fixed $k_\perp = 0.8$ MeV $\hat{e}_x$.

## Results and Discussion

The Wigner distributions of unpolarized and longitudinally polarized physical electron:

$$\begin{aligned}\rho_{UU}(\mathbf{b}_\perp, \mathbf{k}_\perp) &= \frac{4e^2}{2(2\Pi)^2 16\Pi^3} \int d\Delta_x d\Delta_y \int dx \\ &\times\ cos(\Delta_x b_x + \Delta_y b_y) \Big[\frac{1+x^2}{x^2(1-x)^2} \\ &\times\ \Big(\mathbf{p}_\perp^2 - \frac{(1+x)^2}{4}\mathbf{\Delta}_\perp^2\Big) \\ &+\ (M - \frac{m}{x})^2\Big] \varphi'^\dagger(\mathbf{p}''_\perp)\varphi'(\mathbf{p}'_\perp),\end{aligned}$$

$$\begin{aligned}\rho_{LU}(\mathbf{b}_\perp, \mathbf{k}_\perp) &= \frac{4e^2}{2(2\Pi)^2 16\Pi^3} \int d\Delta_x d\Delta_y \\ &\times \int dx \quad sin(\Delta_x b_x + \Delta_y b_y) \\ &\times \frac{x^2-1}{x^2(1-x)} \Big(\Delta_x p_y - \Delta_y p_x\Big) \\ &\quad \varphi'^\dagger(\mathbf{p}''_\perp)\varphi'(\mathbf{p}'_\perp),\end{aligned}$$

where

$$\begin{aligned}\mathbf{p}'_\perp &= \mathbf{p}_\perp - (1-x)\frac{\mathbf{\Delta}_\perp}{2}, \\ \mathbf{p}''_\perp &= \mathbf{p}_\perp + (1-x)\frac{\mathbf{\Delta}_\perp}{2}.\end{aligned}$$

The obtained results are plotted in Fig.(1) and Fig.(2) in impact parameter space as well as in momentum space. The distributions $\rho_{UU}$ are circularly symmetric in both the impact-parameter and the momentum space. The peak of distribution in the momentum space is however in negative direction. The distributions $\rho_{LU}$ show a dipolar behaviour in impact-parameter space as well as in momentum space.

(a)

(b)

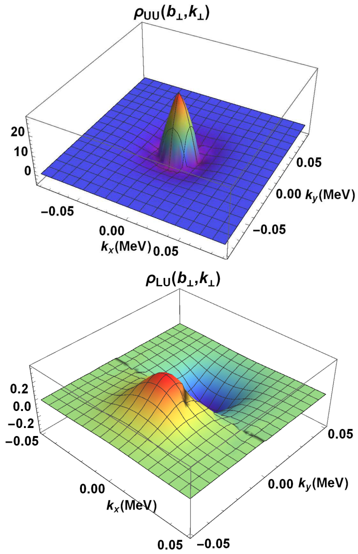


FIG. 2: Wigner distributions $\rho_{LU}(b_\perp, p_\perp)$ and $\rho_{UU}(b_\perp, k_\perp)$ in momentum space at fixed $b_\perp = 0.8$ MeV $\hat{e}_x$.

## References

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