Mass Spectrum of D_s Meson in a Non Relativistic Quark Model with Hulthen Potential

E Deepak D'Silva[1],* A. P. Monteiro[1], Praveen P. D'Souza[1 2], Vipin Naik N. S.[1 2], and K. B. Vijaya Kumar[2]
[1]P.G. Department of Physics, St Philomena College, Darbe, Puttur-574202,INDIA and
[2]Deptartment Physics, Mangalore University, Mangalagangothri P.O., Mangalore - 574199, INDIA

Introduction

D mesons are particles made of a charm and a lighter antiquark or an anticharm and a lighter quark. They are short-lived and decay in multiple ways, which makes them an interesting object of research. Recently significant experimental progress has been achieved in studying the spectroscopy of mesons with one heavy (Q=c,b) and one light (q=u,d,s) quarks [1]. The D_s meson is a light-heavy quark structure composed of the charm and the strange quark. The ground state masses as well as the 1P state masses of the Ds meson have been measured quite accurately. Recently some of the higher excited states such as the $D_{s1}(2710)$, D_{sJ} (2860) and $D_{sJ}(2040)$ have been experimentally measured. The recent experimental observations of open flavour mesonic states at charm sector have provided a boost to the theoretical attempts towards the understanding of the dynamics of light quarks in the company of heavy flavour quarks [2]. In this paper we make an attempt to study the property, which is mass spectrum of the D_s meson based on a non-relativistic quark model with Hulthen Potential.

Theoretical Background

The Hamiltonian employed in our model, includes kinetic energy part, confinement potential and Hulthen potential.

$$H = K + V_{CONF} + V_H \qquad (1)$$

The kinetic energy part (K) is the sum of the kinetic energies including the rest mass minus the kinetic energy of the center of mass motion (CM) of the total system, i.e., The confinement potential must come ultimately from a non-perturbative treatment of QCD. In phenomenological quark models the confinement potential is assumed to be harmonic oscillator potential ($V \sim r^2$) or logarithmic potential ($V \sim ln(r)$) or linear potential ($V \sim r$). For our model we have chosen the linear potential which represents the non perturbative effect of QCD that confines quarks within the color singlet system [3].

$$V_{CONF}(\vec{r}_{ij}) = -a_c r_{ij} \vec{\lambda}_i \cdot \vec{\lambda}_j \qquad (2)$$

where a_c is the confinement strength and λ_i and λ_j are the generators of the color SU(3) group for the i^{th} and j^{th} quarks. It should be noted that the two body confinement potential, has symmetric and antisymmetric terms. The Hulthen potential is one of the important short-range potentials which behaves like Coulomb potential for small values of r and decreases exponentially for large values of r [4].The Hulthen potential V_H is defined as the

$$V_H(\vec{r}) = -\mu_0 \frac{\exp(\frac{-r}{\mu})}{1 - \exp(\frac{-r}{\mu})} \qquad (3)$$

Where μ_0 is a constant and μ is the screening parameter, determining the range for Hulthen potential. The Hulthen potential displays a typical property of the screening effect of a Coulomb-type interaction near the origin($r \to 0$), but it approaches to zero exponentially in the asymptotic region for $r \to \infty$. Hence in the limit $r \to 0$ the Hulthen potential behaves like coulomb -like potential with the strong coupling constant α_s is given by $V_H \simeq \frac{-4\alpha_s}{3r}$. Where α_s is the running coupling constant. The spin dependent potential V_{SD} is introduced as an additional term to the potential to take into the account the spin-orbit and spin-spin interactions, causing the split-

*Electronic address: deepak.dsilva@gmail.com

ting of the nL levels (n is the principal quantum number, L is the orbital momentum), so it has the form [5].

$$V_{SD}(r) = \left(\frac{L\cdot S_c}{2m_c^2} + \frac{L\cdot S_b}{2m_b^2}\right)\left(-\frac{dV(r)}{rdr} + \frac{8}{3}\alpha_s\frac{1}{r^3}\right) + \frac{4}{3}\alpha_s\frac{1}{m_c m_b}\frac{L\cdot S}{r^3} + \frac{4}{3}\alpha_s\frac{2}{3m_c m_b}S_c\cdot S_b 4\pi\delta(r) + \frac{4}{3}\alpha_s\frac{1}{3m_c m_b}\left[3(S_c\cdot n)(S_b\cdot n) - S_c\cdot S_b\right]\frac{1}{r^3}$$

The non−central part of OGEP has two terms, namely the spin−orbit interaction $V^{SO}_{OGEP}(\vec{r})$ and tensor term $V^{ten}_{OGEP}(\vec{r})$. The spin-orbit interaction of OGEP is given by,

$$V^{SO}_{OGEP}(\vec{r}) = -\frac{\alpha_s}{4}\lambda_{\mathbf{i}}\cdot\lambda_{\mathbf{j}}\left[\frac{3}{8M_iM_j}\frac{1}{r^3}(\vec{r}\times\vec{p})\cdot(\sigma_{\mathbf{i}}+\sigma_j)\right]$$

where the relative angular momentum is defined as usual in terms of relative position $\vec{r}$ and the relative momentum $\vec{p}$. Unlike the tensor force, the spin−orbit force does not mix states of different $\vec{L}$, since L^2 commutes with $\vec{L}\cdot\vec{S}$, $\vec{L}$ is still a constant of motion, but L_z is not. We use the following tensor term.

$$V^{ten}_{OGEP}(\vec{r}) = -\frac{\alpha_s}{4}\lambda_{\mathbf{i}}\cdot\lambda_{\mathbf{j}}\left[\frac{1}{4M_iM_j}\frac{1}{r^3}\right]\hat{S}_{ij}$$

$$\hat{S}_{ij} = [3(\vec{\sigma}_i\cdot\hat{r})(\vec{\sigma}_j\cdot\hat{r}) - \vec{\sigma}_i\cdot\vec{\sigma}_j]$$

The tensor potential is a scalar which is obtained by contracting two second rank tensors. Here, $\hat{r} = \hat{r}_i - \hat{r}_j$ is the unit vector in the direction of $\vec{r}$. In the presence of the tensor interaction, $\vec{L}$ is no longer a good quantum number.

Results and Discussions

We have solved the Schrodinger equation with Hamiltonian equation using the variational method:

$$E(\psi) = \frac{\langle\psi|H|\psi\rangle}{\langle\psi|\psi\rangle} = \langle H\rangle$$

In our work, we have used the three-dimensional harmonic oscillator wave function The results obtained agree with the experimental results and with the theoretical predictions from other models.

Table 1. The mass spectrum of D_s meson

$n^{2S+1}L_J$	The Mass MeV	M_{exp} MeV	[2] MeV
$D_s(1^1S_0)$	1968.41	1968.49±0.32	1979
$D_s(2^1S_0)$	2632.67	2632.5±1.7	2673
$D_s(3^1S_0)$	3179.25	....	3154
$D_s(1^3S_1)$	2112.17	2112.3±0.5	2129
$D_s(2^3S_1)$	2724.86	2710^{+12}_{-7}	2732
$D_s(3^3S_1)$	3219.14	....	3193
$D_s(1^1P_1)$	2461.54	2459.6±0.6	2549
$D_s(2^1P_1)$	3061.25	....	3018
$D_s(3^1P_1)$	3525.00	....	3416
$D_s(1^3P_0)$	2318.40	2317.8±0.6	2484
$D_s(2^3P_0)$	2919.26	....	3005
$D_s(3^3P_0)$	3326.96	....	3412
$D_s(1^3P_1)$	2536.09	2535.12±0.13	2556
$D_s(2^3P_1)$	3044.91	3044^{+30}_{-9}	3038
$D_s(3^3P_1)$	3471.07	....	3433
$D_s(1^3P_2)$	2572.18	2571.9±0.8	2592
$D_s(2^3P_2)$	3128.43	....	3048
$D_s(3^3P_2)$	3588.01	....	3439
$D_s(1^1D_2)$	2878.85	....	2900
$D_s(2^1D_2)$	3474.41	....	3298
$D_s(3^1D_2)$	3820.56	....	3650
$D_s(1^3D_1)$	2854.17	....	2899
$D_s(2^3D_1)$	3298.27	....	3306
$D_s(3^3D_1)$	3808.72	....	3661
$D_s(1^3D_2)$	2780.04	....	2926
$D_s(2^3D_2)$	3340.64	....	3323
$D_s(3^3D_2)$	3835.28	....	3672
$D_s(1^3D_3)$	2863.70	2862^{+6}_{-3}	2917
$D_s(2^3D_3)$	3354.76	....	3311
$D_s(3^3D_3)$	3858.89	....	3658

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References

[1] D. Ebert, R. N. Faustov, V. O. Galkin. The Eur.Phy.Journal C. **66**, 197-206 (2010)

[2] Virendrasinh H. Kher, Nayneshkumar Devlani and Ajay Kumar Rai. EPJ Web of Conferences. **95**, 05005 (2015)

[3] A. P Monteiro, Manjunath Bhat and K. B. Vijaya Kumar, International Journal of Modern Physics A **32**, 04,(2017)

[4] Bhaghyesh and K. B. Vijaya Kumar and A. P. Monteiro, J.of Phy. G, 38 **8** (2011)

[5] A. P. Monteiro, Manjunath Bhat and K. B. Vijaya Kumar. Physics Review D, 054016 **95** (2017)