Instatnon Contribution to the charmonium Mass spectra

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Introduction

Our Model uses the NRQM formalism for the study of properties of charmonium states using a Hamiltonian which has the heavy quark potential derived from the instanton vacuum depending on r, the inter quark distance. The heavy quark potential derived from the instanton ensemble rises linearly as the relative distance between the quark and anti quark increases and gets saturated. As the quark and the antiquark distance increases the central potential turns out to be Coulomb like potential. In this work, we have investigated the instanton effects on the heavy-quark static potential. We have discussed the instanton effects on the masses of charmonia and hyperfine mass splitting [3]. The existence of the instanton gas does not lead to quark confinement. It should be noted that the instantons will not give linearly rising potential at large distances, hence it cannot explain the confinement of quark-anti quark. Hence, we have included linear confinement potential in the Hamiltonian for confinement as predicted by the lattice gauge theory.

Theoretical Background

In a potential model approach the entire dynamics of quarks in a meson is governed by a Hamiltonian has kinetic energy term (K) and a potential energy (V), that is,

$$H = K + V.$$
$$K = M + \frac{p^2}{2\mu}$$

Here p is the relative momentum, $\mu = \frac{m_Q m_{\bar{Q}}}{m_Q + m_{\bar{Q}}}$ is the reduced mass of the $Q\bar{Q}$ system, where m_Q and $m_{\bar{Q}}$ are the masses of the individual quark and anti quark respectively and M is the total mass of quark and antiquark [2].

The potential energy V is the sum of the heavy-quark potential $V_{Q\bar{Q}}(\vec{r})$,confining potential $V_{conf}(\vec{r})$ and Coulomb potential $V_{coul}(\vec{r})$.

$$V(\vec{r}) = V_{Q\bar{Q}}(\vec{r}) + V_{coul}(\vec{r}) + V_{conf}(\vec{r})$$
$$V_{Q\bar{Q}}(\vec{r}) = V_C(\vec{r}) + V_{SD}(\vec{r}).$$

Here $V_C(\vec{r})$ and $V_{SD}(\vec{r})$ are central and spin dependent potentials due to instanton vacuum respectively [3].
$V_C(\vec{r})$ is given by the following expression

$$V_C(\vec{r}) \simeq \frac{4\pi\bar{\rho}^3}{\bar{R}^4 N_c}\left(1.345\frac{r^2}{\bar{\rho}^2} - 0.501\frac{r^4}{\bar{\rho}^4}\right) \quad (1)$$

Here, $\bar{\rho} = \frac{1}{3}$ fm the average size of the instanton, $\bar{R} = 1$ fm the average separation between instantons and number of colors N_C is 3.

The spin-spin interaction $V_{SS}(\vec{r})$, the spin-orbit coupling term $V_{LS}(\vec{r})$ and the tensor part $V_T(\vec{r})$ contribute to the spin dependent potential $V_{SD}(\vec{r})$;

$$V_{SD}(\vec{r}) = V_{SS}(\vec{r}) + V_{LS}(\vec{r}) + V_T(\vec{r})$$
$$V_{SS}(\vec{r}) = \frac{1}{3{m_Q}^2}\nabla^2 V_C(\vec{r}); \quad V_{LS}(\vec{r}) = \frac{1}{2{m_Q}^2}\frac{1}{r}\frac{dV_C(\vec{r})}{dr};$$
$$V_T(\vec{r}) = \frac{1}{3{m_Q}^2}\left(\frac{1}{r}\frac{dV_C(\vec{r})}{dr} - \frac{d^2 V_C(\vec{r})}{dr^2}\right).$$

The coulomb-like (perturbative) one gluon exchange part of the potential is given by

$$V_{coul}(\vec{r}) = \frac{-4\alpha_s}{3r} \quad (2)$$

with the strong coupling constant α_s and inter quark distance r. The confinement term represents the non perturbative effect of QCD which includes the spin-independent linear confinement term [39].

$$V_{conf}(\vec{r}) = -\left[\frac{3}{4}V_0 + \frac{3}{4}cr\right]F_1 \cdot F_2 \quad (3)$$

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where c and V_0 are constants. F is related to the Gell-Mann matrix, $F_1 = \frac{\lambda_1}{2}$ and $F_2 = \frac{\lambda_2^*}{2}$ and $F_1{\cdot}F_2 = \frac{-4}{3}$ for the mesons. In our work, we have used the three-dimensional harmonic oscillator wave function which has been extensively used in atomic and nuclear physics is used as the trial wave function for obtaining the $Q\bar{Q}$ mass spectrum.

$$\psi_{nlm}(r,\theta,\phi) = N\left(\frac{r}{b}\right)^l L_n^{l+1/2}(\frac{r}{b}) exp\left(-\frac{r^2}{2b^2}\right) Y_{lm}(\theta,\phi)$$

where $|N|$ is the normalizing constant given by

$$|N|^2 = \frac{2\alpha^3 n!}{\sqrt{\pi}} \frac{2^{(2(n+l)+1)}}{(2n+2l+1)!}(n+l)! \quad (4)$$

and $L_n^{l+1/2}(x)$ are the associated Laguerre polynomials,

Results and Discussions

It is found that the instanton effects on the quarkonia spectra are required for quantitative description of both mass spectra and their decay rates. Instanton contributions are small but non negligible. More heavier the quark mass, the effects from instantons become less. Also, it has been shown that re scattering of gluons with instantons generate effective mass for gluons which is attributed to nonperturbative effects arising from instanton vacuum [1]. Hence, nonperturbative effects from instanton vacuum has to be considered. We have investigated in this work the instanton effects on the heavy-quark static potential and the instanton contributions on the masses of charmonia are as shown in the table 2. The potential from the instanton vacuum is sensitive to the instanton parameters. The instanton effects come play significantly in mass spectrum of charmonium, they turn out to be rather small in describing the hyperfine mass splittings of the charmonia. This might be due to the spin dependent part of the potential from the instanton vacuum being almost on order of magnitude smaller the central part. The tensor interaction contributes almost nothing to the result. The inclusion of the Coulomb like potential coming from the pertubative one gluon exchange and linear confining potential to the instanton potential, shows the significant result in hyperfine mass splittings and mass splittings of charmonium.

Table 1 Instanton effects on the hyperfine mass splitting (MeV)

Mass Splittings	Present Work	Exp
$M(1^3S_1 - 1^1S_0)$	113	113.2 ± 0.7
$M(2^3S_1 - 2^1S_0)$	47	47 ± 1
$M(1^3P_2 - 1^3P_1)$	45	45.7 ± 0.2
$M(1^3P_1 - 1^3P_0)$	96	95.5 ± 0.8

Table 2 The contributions from instantons(M_I) to the charmonia (MeV)

$n^{2S+1}L_J$	Name	The Mass MeV	M_{exp} MeV	M_I MeV
1^1S_0	$\eta_c(1S)$	2984	2983.6±0.7	16.39
1^3S_1	J/ψ	3097	3096.916±0.011	14.92
2^1S_0	$\eta_c(2S)$	3640	3639.2±0.11	17.91
2^3S_1	$\psi(2S)$	3687	3686.108±0.018	17.84
3^1S_0	$\eta_c(3S)$	4061	....	17.41
3^3S_1	$\psi(3S)$	4039	4039± 1	17.08
1^3P_0	$\chi_{c0}(1P)$	3414	3414.75±0.31	7.41
1^3P_1	$\chi_{c1}(1P)$	3510	3510.66±0.07	6.91
1^1P_1	$h_c(1P)$	3525	3525.38±0.11	6.14
1^3P_2	$\chi_{c2}(1P)$	3555	3556.20±0.09	6.77
2^3P_0	$\chi_{c0}(2P)$	3916	$3915{\pm}3 \pm 2$	12.09
2^3P_1	$\chi_{c1}(2P)$	3872	3872	12.54
2^1P_1	$h_c(2P)$	3927	....	11.84
2^3P_2	$\chi_{c2}(2P)$	3929	3927.2±2.6	12.24
1^3D_3	$\psi_2(1D)$	3845	....	5.33
1^1D_2	$\eta_{c2}(1D)$	3812	...	4.62
1^3D_2	$\psi_2(1D)$	3823	3823	5.32
1^3D_1	$\psi_1(1D)$	3779	3778	5.42
2^3D_3	$\psi_2(2D)$	4220	....	10.30
2^1D_2	$\eta_{c2}(2D)$	4198	....	9.97
2^3D_2	$\psi_2(2D)$	4195	....	10.33
2^3D_1	$\psi_1(2D)$	4192	4191±5	10.23
3^3D_3	$\psi_2(3D)$	4581	....	14.61
3^1D_2	$\eta_{c2}(3D)$	4550	....	14.50
3^3D_2	$\psi_2(3D)$	4561	....	14.62
3^3D_1	$\psi_1(3D)$	4520	....	14.69

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