Mass spectral calculations of pseudoscalar and vector charmonia in framework of Bethe-Salpeter equation using algebraic approach

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Abstract: In this work we calculate the mass spectrum of 1S,...,5S states of 0^{-+} , and 1^{--} $c\bar{c}$ systems [1] in the framework of a QCD motivated Bethe-Salpeter Equation. In this 4 x 4 BSE framework [2-4], the coupled Salpeter equations are first shown to decouple for the confining part of interaction, under heavy-quark approximation, and analytically solved, and later the one-gluon-exchange interaction is perturbatively incorporated, leading to mass spectral equations for various quarkonia. In this calculation we did an exact treatment of the spin part of the interaction kernel [3], which is a substantial improvement over our earlier work [2]. The analytic forms of wave functions obtained can be used for calculation of decay widths of various processes (see [2,3]). Our results are in reasonable agreement with data (where ever available) and other models [2].

1.Introduction: There has been a renewed interest in recent years in spectroscopy of $Q\bar{Q}$ in charm and beauty sectors, which was primarily due to experimental facilities the world over such as BABAR, Belle, CLEO, DELPHI, BES etc. [1], which have been providing accurate data on $c\bar{c}$ and $b\bar{b}$ hadrons with respect to their masses and decays. These conventional heavy $Q\bar{Q}$states such as $c\bar{c}$ and $b\bar{b}$ are well understood to be quark- anti quark pair bound by the coulombic potential, that arises due to the perturbative one-gluon-exchange that dominates its short range part, and the linearly rising part that describes the confining potential at long distances, and are crucially important to improve our understanding of QCD. We do understand that in $Q\bar{Q}$ quarkonia, the constituents are close enough to each other to warrant a more accurate treatment of the OGE (Coulomb) term, but it may not be so unreasonable to treat the coulomb term perturbatively for charmonium ($c\bar{c}$) systems [2]. And if we are getting reasonable results for orbital $c\bar{c}$ excitations, it is mainly due to the centrifugal effects that ensure that $c-\bar{c}$ separation is large enough to feel the effect of confining term more strongly than the coulomb term.

2. Calculation: We start with the 4D form of Bethe-Salpeter equation, which using a sequence of steps outlined in [2], are reduced to a set of four Salpeter equations (in 4D variable, $\hat{q}$) under Covariant Instantaneous Ansatz (CIA)- which is a Lorentz-invariant generalization of Instantaneous Approximation (IA). These are covariant forms of Salpeter equations which are valid for hadrons in arbitrary motion. Thus, in the present paper, the coupled Salpeter equations for pseudoscalar (0^{-+}), and vector (1^{--}) quarkonia are first shown to decouple for the confining part of interaction, under heavy-quark approximation, and the analytic forms of mass spectral equations are worked out, which are then solved in approximate harmonic oscillator basis to obtain the unperturbed wave functions for various states of these quarkonia. We then incorporate the one-gluon-exchange perturbatively into the unperturbed spectral equation, and obtain the full spectrum. The wave functions of scalar (0^{-+}) and axial vector (1^{--}) quarkonia, are also calculated [3]. To solve these equations, we need the BS kernel, which is taken to be one-gluon exchange like as regards the colour and spin dependence, while the potential V($\hat{q},\hat{q}'$) involves the scalar structure of the gluon propagator in the perturbative (o.g.e), as well as the non-perturbative (confinement) regimes. Here we did an exact treatment of the spin structure of the kernel, that appears as, $\gamma_\mu\psi(P,q)\gamma_\mu$ on the right side of the Salpeter equations, where we used the full Dirac structure of the Dirac wave function, $\psi(P,q)$, which lead to a set of coupled integral equations in the amplitudes associated with various Dirac structures. The resulting eigen value equation obtained after decoupling the Salpeter equations and perturbative incorporation of the coulomb term is

$$E_{P,V}\phi_{P,V}(\hat{q}) = (-\beta_{P,V}^4\vec{\nabla}_q^4 + \hat{q}^2 + V_{Coul.}^{P,V})\phi_{P,V}(\hat{q}), \quad (1)$$

which leads to the complete mass spectral equation

$$\left(\frac{M^2}{4} - m^2 + \frac{\beta_{P,V}^4 C_0}{\omega_0^2}\right) + 2\beta_{P,V}^2\gamma < V_{coul.}^{P,V} > = 2\beta_{P,V}^2(N + \tfrac{3}{2})\ ;$$

$$N = 2n + l\ ; n = 0,1,2,..., \quad (2)$$

where $\beta_{P,V}$ is the inverse range parameter for P and V-mesons respectively. The analytic forms of the unperturbed wave functions for 1S,…,5S states of pseudoscalar (P) and vector (V) quarkonium that are obtained as algebraic solutions of Eqs.(1) (without the coulomb term, $V_{Coul.}^{S,A}$ are given in [3]. The expectation value of the coulomb term between the various unperturbed nS states for pseudoscalar mesons, and between the unperturbed nS and nD states of vector mesons being,

$$< nS|V_{coul.}^{P}|nS >= \frac{\pi\alpha_s}{12\beta_P^2}$$

$$< nS|V_{coul.}^{V}|nS >= \frac{\pi\alpha_s}{24\beta_V^2}$$

$$< nD|V_{coul.}^{V}|nD >= \frac{\pi\alpha_s}{120\beta_V^2} \quad (3)$$

The non-zero values of $< V^{coul.} >$ given above, not only lead to the lifting up of the degeneracy between the S and D levels with the same principal quantum number N in vector quarkonia, but also lead to bringing the masses of different states of vector and pseudoscalar quarkonia closer to data.

3. Results and Conclusions: It is observed that the mass spectra of mesons of various J^{PC} is somewhat insensitive to a range of variations of parameter $\omega_0\epsilon$[0.130- 0.160] GeV.,as long as C_0/ω_0^2 is a constant, and reasonably good fits are obtained for $\omega_0 = 0.145\ GeV.$, C_0 =0.15022, and charmed quark mass, $m_c = 1.490\ GeV.$[2]. The mass spectral predictions for 1S,…,5S states for 0^{-+}, and 1^{--} states are given in Tables I and II respectively.

	BSE-CIA	Expt.[1]
$\eta_c(1S)$	2.9759	2.9839 ± 0.0005
$\eta_c(2S)$	3.7264	3.6376 ± 0.0012
$\eta_c(3S)$	4.4812	
$\eta_c(4S)$	5.1368	
$\eta_c(5S)$	5.7442	

Table I: Mass spectral predictions (in GeV.) of pseudoscalar (0^{-+}) charmonium for 1S,…,5S states, along with experimental results.

	BSE-CIA	Expt.[1]
$J/\psi(1S)$	3.1017	3.0969 ± 0.00001
$\psi(2S)$	3.6854	3.6861 ± 0.00025
$\psi(1D)$	3.7011	3.773 ± 0.00035
$\psi(3S)$	4.2154	4.0390 ± 0.0001
$\psi(2D)$	4.2518	4.1910 ± 0.005
$\psi(4S)$	4.7044	4.421 ± 0.004
$\psi(3D)$	4.7628	
$\psi(5S)$	4.9939	

Table II: Mass spectral predictions (in GeV.) of vector (1^{--}) charmonium, for 1S,…,5S states, along with experimental results.

We have given the plots of our wave functions derived analytically for states 1S,…, 5S for pseudoscalar and vector mesons (given in Ref. [2]) are very similar to the corresponding plots of wave functions obtained using purely numerical approaches of solving coupled integral equations. The transparency of our purely algebraic treatment that takes all Dirac structures in hadronic BS wave functions, gives more insight into the mass spectral problem than the purely numerical approaches prevalent in the literature.

4. References:

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