Radiative decays of charmonia in the framework of Bethe-Salpeter equation

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Abstract: In this work, we study the two-photon decay widths of a scalar charmonium, 0^{++}, and the single photon radiative decay widths of vector charmonia, 1^{--} through the quark triangle diagrams in Figs.1, and 2 respectively, using the framework of Bethe-Salpeter equation. We have compared our results with data and other models.

1. Introduction: Spectroscopy of heavy hadrons in charm and beauty sectors, has emerged as a frontier area of research these days, primarily due to experimental facilities the world over such as BABAR, Belle, CLEO, DELPHI, BES etc. [1], which have been providing accurate data on $c\bar{c}$ and $b\bar{b}$ hadrons with respect to their masses and decays. The conventional heavy $Q\bar{Q}$states such as $c\bar{c}$ and $b\bar{b}$ are well understood to be quark- anti quark pair bound by the coulombic potential, that arises due to the perturbative one-gluon-exchange, that dominates its short range part, and the linearly rising part that describes the confining potential at long distances, and are crucially important to improve our understanding of QCD.

2. Two photon decays of a scalar quarkonium: can be described by the famous quark- triangle diagram shown in Fig.1. Let P be the total momentum of the scalar quarkonia, and $k_{1,2}$ be the momenta of the two emitted photons with polarizations $\varepsilon_{1,2}$respectively. Then we can write, $P = k_1 + k_2$, and $2Q = k_1 - k_2$, where $2Q$ is the relative momentum of the emitted photons. The invariant amplitude for this process can be written as,

$$M_{fi} = \frac{i\sqrt{3}e_q^2}{m^2 + \frac{M^2}{4}} \int \frac{d^3q}{(2\pi)^3} Tr\{\psi^S(\hat{q}) \times$$

$$[\gamma.\varepsilon_1(m_1 + i\gamma.Q)\gamma.\varepsilon_2 + \gamma.\varepsilon_2(m_1 + i\gamma.Q)\gamma.\varepsilon_1]\} \quad (1)$$

where $e_q = +\frac{2}{3}e$ for $c\bar{c}$ mesons. The 3D structure of BS wave function, $\psi^S(\hat{q})$ is given as [2],

$$\psi^S(\hat{q}) = [M + i\frac{Mm\gamma.\hat{q}}{\hat{q}^2} - 2\frac{\gamma.P\gamma.q}{M}]\phi^S(\hat{q}) \,, \quad (2)$$

which arises from the reduction of the 4D BS wave function [4] to 3D form under Covariant Instantaneous Ansatz. The decay width for the process can be expressed as,

$$\Gamma(S \to \gamma\gamma) = \frac{|F_S|^2}{32\pi M} \,, \quad (3)$$

with the decay constant expressed as,

$$F_S = \frac{16\alpha_{e.m.}}{3\sqrt{3}\,\pi^2} \frac{mM}{m^2 + \frac{M^2}{4}} N_S \int d^3\hat{q}\, \phi^S(\hat{q}) \quad (4)$$

where $\phi^S(\hat{q})$ is the 3D BS wave function [2] for the scalar charmonium given in Eq.(2), N_S being the 4D BS normalizer for this meson, M being the hadron mass, and m is the constituent mass for the charmed quark.

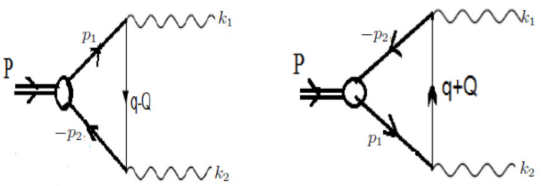

Fig.1: Feynman diagrams contributing to two-photon decays of scalar charmonium.

We obtain the decay widths, $\Gamma_{\chi_{c0}(1P)\to\gamma\gamma} = 3014 eV.$ $(Expt. = 2341.50 eV.)$, and $\Gamma_{\chi_{c0}(2P)\to\gamma\gamma} = 1910.00 eV.$

3. Radiative decays of charmonia: We have studied the single photon radiative decay widths of vector charmonia for the process, J/$\psi \rightarrow \eta_c(nS)\gamma$, for principal quantum numbers, $n = 1,2,\dots,4$. The lowest order Feynman diagrams are given in Fig.2, where the second diagram is obtained from the first diagram by reversing the direction of internal fermion lines.

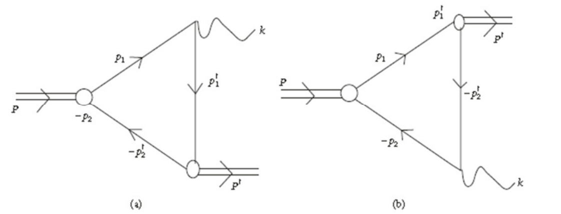

Fig.2: Single photon radiative decays of vector charmonia

The invariant amplitude for this process is

$$M_{fi} = -ie_q \int \frac{d^4q}{(2\pi)^4} Tr[\bar{\psi}_P(P',q')\gamma.\varepsilon\psi_V(P,q)S_F^{-1}(-p_2) + \bar{\psi}_P(P',q')S_F^{-1}(p_1)\psi_V(P,q)\gamma.\varepsilon] , \quad (5)$$

where the second term gives the same contribution as the first term. Here $\psi_P(P',q')$ and $\psi_V(P,q)$ are the 4D BS wave functions of pseudoscalar and vector charmonia, and contain all the Dirac structures for pseudoscalar and vector mesons of external momenta P', and P, and internal momenta q' and q respectively, and p_1, p'_1 $(-p_2, -p'_2)$ are the momenta of the quarks (anti-quarks) respectively for the two hadrons, and k is the momentum of the emitted photon of polarization, ε. The BS wave functions of 0^{-+}, and 1^{--}states after relativistic 3D reduction under CIA contains a combination of Dirac structures, and have the forms,

$$\psi_P(\hat{q}') = N_P'\left[M' + P' + \frac{(\gamma.\hat{q}')(\gamma.P')}{m}\right]\gamma_5\phi_P(\hat{q}'),$$

$$\psi_V(\hat{q}) = N_V[M\gamma.\epsilon + \hat{q}.\epsilon\frac{M}{m} + \gamma.\epsilon\gamma.P + \frac{\gamma.P\epsilon.\hat{q}}{m} - \frac{\gamma.P\gamma.\epsilon\gamma.\hat{q}}{m}]\phi_V(\hat{q}), \quad (6)$$

respectively. Here, $\phi_V(\hat{q}), \phi_P(\hat{q})$ are the scalar forms of 3D wave functions derived from respective mass spectral equations [2,3] The decay width is expressed as,

$$\Gamma_{V\rightarrow P\gamma} = \frac{|\vec{P'}|}{8\pi M^2}|M_{fi}|^2; \quad (7)$$

$$M_{fi} = f(\hat{q})\epsilon_{\mu\nu\alpha\beta}P_\mu P_\nu'\varepsilon_\alpha\epsilon_\beta ;$$

$$f(\hat{q}) = 2e_q N_P' N_V \int \frac{d^3\hat{q}}{(2\pi)^3}\phi_P(\hat{q'})\phi_V(\hat{q}) \times \left[\frac{m}{\omega'^2} + \frac{1}{\omega'}\right][\frac{m^3}{\omega^2} - \frac{m^2}{\omega} + (\frac{m}{\omega^2} - \frac{1}{\omega})\hat{q}.\hat{q}'],$$

where $F(\hat{q})$ is the form factor. Radiative decay widths, Γ for nS states of J/ψ of this BSE framework and comparison with data are given in Table 1 below.

	BSE-CIA	Expt.[1]
J/$\psi(1S) \rightarrow \eta_c(1S)\gamma$	2.0054	1.5687±0.011
$\psi(2S) \rightarrow \eta_c(2S)\gamma$	0.5709	0.2093±0.002
$\psi(3S) \rightarrow \eta_c(3S)\gamma$	0.2984	

Table 1: Radiative decay widths, Γ for 1S,…,3S states of J/ψ.

The aim of doing this study was to do a purely analytic treatment of 4 ×4 BSE for mass spectra as well as various transition amplitudes [2-4], and test our analytic forms of wave functions, $\phi_S(\hat{q})$, $\phi_P(\hat{q})$ and $\phi_V(\hat{q})$ in [2-3] as solutions of mass spectral equations in an approximate harmonic oscillator basis obtained analytically as solutions of BSE. The transparency of our approach gives a much more deeper understanding of the underlying quark dynamics of heavy hadrons.

4. References:

[1] M.Tanabashi et al., (Particle Data Group), Phys. Rev. D98, 030001 (2018).

[2] S.Bhatnagar, L.Alemu, Phys. Rev. D97, 034021 (2018), and references therein.

[3] H.Negash, S.Bhatnagar, Intl. J. Mod. Phys. E25, 1650059 (2016).

[4] H.Negash, S.Bhatnagar, Adv. in HEP, 2017, 7306825 (2017).