Analyzing Powers for $\vec{p}\vec{p} \to p\Delta^+$

Venkataraya[1,2,3] Sujith Thomas[1] G. Ramachandran[1,4]
[1]G.V.K. Academy, BSK II Stage, Bengaluru - 560070, India
[2]Vijaya College, Jayanagar, Bengaluru - 560011, India
[3]Amrita Vishwa Vidyapeetham, Coimbotore-641112, India
[4]Amrita Vishwa Vidyapeetham, Kasavanahalli, Bengaluru-560035, India
e-mail: venkataraya@gmail.com;sogiththomas@gmail.com; gwrvrm@yahoo.com.

Introduction

The theoretical study of neutral pion production in proton-proton collisions has excited considerable attention since the experimental measurements for the total cross-section were found to be more than five times larger than the then existing theoretical predictions. Advances in technologies not only led to the complete identification of the three-body final state kinematically, but also to measurements employing polarized beam on a polarized target [1]. Hanhart et.al., [2] observed: " As far as microscopic model calculations of the reaction $NN \to NN\pi$ are concerned, one has to concede that the theory is definitely lagging behind the development of experimental sector". The Julich meson exchange model [2] which was comparatively more successful with the less complete data on $\vec{p}\vec{p} \to d\pi^+$ and $\vec{p}\vec{p} \to np\pi^+$, failed to provide an overall satisfactory reproduction of the complete set of polarization observables in the case of $\vec{p}\vec{p} \to np\pi^0$. A model independent approach [3] using the irreducible tensor techniques [4] was developed to analyse the data. A comparison [5] was made to incisively analyse the findings of the Julich model which not only identified some of the deficiencies in the model but also revealed the importance of Δ contributions. A phase ambiguity inherent in [5] was pointed out in [6].

The purpose of this contribution is to present a model independent theoretical study of the analyzing powers in $\vec{p}\vec{p} \to p\Delta^+$. We supplement the analysing powers to the unpolarized differential cross-section in Δ production amplitudes.

Theoretical formalism

In an experiment like [1], where both the beam and target are polarized, the initial polarization state is described by the spin density matrix

$$\rho^i = \frac{1}{4}(1 + \boldsymbol{\sigma} \cdot \boldsymbol{P}) \cdot (1 + \boldsymbol{\sigma} \cdot \boldsymbol{Q}) \qquad (1)$$

We express ρ^i as

$$\rho^i = \frac{1}{4} \cdot \sum_{k_1,k_2=0}^{1} (\sigma^{k_1}(1) \cdot P^{k_1}) \cdot (\sigma^{k_2}(2) \cdot Q^{k_2})$$

$$= \frac{1}{4} \sum_{k_1,k_2=0}^{1} \sum_{k=|k_1-k_2|}^{k_1+k_2} (-1)^{k_1+k_2-k} (\sigma^{k_1} \otimes \sigma^{k_2})\,(P^{k_1} \otimes Q^{k_2}) \qquad (2)$$

introducing the notations

$$P_0^0 = 1;\; P_0^1 = P_z;\; P_{\pm1}^1 = \mp \frac{P_x \pm iP_y}{\sqrt{2}}$$

$$Q_0^0 = 1;\; Q_0^1 = Q_z;\; Q_{\pm1}^1 = \mp \frac{Q_x \pm iQ_y}{\sqrt{2}}$$

$$\sigma_0^0(j) = 1; \sigma_0^1(j) = \sigma_z(j); \sigma_{\pm1}^1(j) = \mp \frac{\sigma_x(j) \pm i\sigma_y}{\sqrt{2}}$$

$$j = 1,2 \qquad (3)$$

The differential cross-section for $pp \to p\Delta^+$ is then given by

$$\frac{d\sigma}{d\Omega_2} = tr\mathrm{M}\rho^i M^\dagger \qquad (4)$$

where $\mathrm{d}\Omega_2$ refers to the solid angle associated with the outgoing proton with cm energy

$$E_2 = (E_{cm}^2 - M_\Delta^2 + M_p^2)/2E_{cm} \qquad (5)$$

where M_p and M_Δ denote the masses of the proton and delta respectively. The reaction matrix M for $pp \to p\Delta^+$ may be written in the form

$$M = \sum_{s_i=0}^{1} \sum_{s=1}^{2} \sum_{\Lambda=|s-s_i|}^{(s+s_i)} (S^\Lambda(s, s_i). M^\Lambda(s, s_i)) \qquad (6)$$

where the irreducible spin tensor operators are defined following [4] and the corresponding irreducible tensor amplitudes $M_\mu^\Lambda(s, s_i)$ are expressible in terms of the partial wave amplitudes $M_{l_2 s; l_i s_i}^j$ which include also all factors dependent on cm energy E_{cm}.

$$M_{\mu}^{\Lambda}(s,s_i) = \sum_{l_i,l_2,j}(-1)^{s_i-j}\frac{[j]^2}{[s]} \cdot W(s_i l_i s l_2; j\Lambda) \times M^{j}_{l_2 s;l_i s_i}\left(Y_{l_2}(\hat{p}_2)\otimes Y_{l_i}(\hat{p}_i)\right)^{\Lambda}_{\mu} \quad (7)$$

Using the eq.(2) and eq.(6) in eq.(4), we have

$$\frac{d\sigma}{d\Omega_2} = \frac{d\sigma_0}{d\Omega_2}\sum_{k_1,k_2=0}^{1}\sum_{k=|k_1-,k_2|}^{(k_1+k_2)}(P^{k_1}\otimes Q^{k_2})\cdot A^{k}(k_1,k_2) \quad (8)$$

where, the unpolarized differential cross-section is of the form

$$\frac{d\sigma_0}{d\Omega_2} = \frac{1}{4}\sum_{s,s_i,\Lambda,\mu}|M_{\mu}^{\Lambda}(s,s_i)|^2 = a + b cos^2\theta_2 \quad (9)$$

The analyzing powers $A_q^k(k_1,k_2)$ are obtained through

$$\frac{d\sigma_0}{d\Omega_2}\cdot A_q^k(k_1,k_2) = B_q^k(k_1,k_2) =$$

$$\frac{1}{2}\sum_{s_i,s_i',s,\Lambda,\Lambda'}(-1)^{\Lambda}(-1)^{k_1+k_2-k}[k_1][k_2][s_i][s_i'][\Lambda][\Lambda'] \times \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & s_i \\ \frac{1}{2} & \frac{1}{2} & s_i' \\ k_1 & k_2 & k \end{Bmatrix}\cdot(M^{\Lambda}(s,s_i)\otimes(M^{\dagger\Lambda'}(s,s_i))_q^k \quad (10)$$

$The\ B_q^k(k_1,k_2)$ can be expressed in the form

$$B_q^k(k_1,k_2) = a_q^k(k_1,k_2) + b_q^k(k_1,k_2)\sin\theta_2 + c_q^k(k_1,k_2)\cos\theta_2 + d_q^k(k_1,k_2)\sin\theta_2\cos\theta_2 + g_q^k(k_1,k_2)cos^2\theta_2 \quad (11)$$

where the coefficients are functions of partial wave amplitudes $M^{j}_{l_2 s_2;l_i s_i}$. If we consider only the first three amplitudes

$$F_1 = M^2_{02;20};$$

$$F_2 = M^0_{11;11};$$

$$F_3 = M^1_{11;11} \quad (12)$$

close to threshold, we can determine their modulus values and the relative phase between F_2 and F_3 using the following equations

$$|F_1|^2 = 64\pi^2\left(a_0^0(0{,}0) - 2g_0^0(0{,}0) - a_2^2(1{,}1) + a_{-2}^2(1{,}1)\right) \quad (13)$$

$$|F_2|^2 = 192\pi^2\left(g_0^0(0{,}0) + a_2^2(1{,}1) - a_{-2}^2(1{,}1)\right) \quad (14)$$

$$|F_3|^2 = \frac{256\pi^2}{3}g_0^0(0{,}0) \quad (15)$$

$$ReF_2F_3^* = 32\pi^2\left(6g_0^0(0{,}0) - 8\alpha_2^2(1{,}1) - 2a_{-2}^2(1{,}1)\right) \quad (16)$$

$$ImF_2F_3^* = \frac{64\pi^2 i}{9+3\sqrt{3}-\sqrt{15}}\left\{\left(14-2\sqrt{3}-\sqrt{15}\right)a_2^2(1{,}1) + \left(91-13\sqrt{3}+\sqrt{15}\right)a_{-2}^2(1{,}1) - \left(99-18\sqrt{3}\right)g_0^0(0{,}0) - 36c_1^1(1{,}0)\right\} \quad (17)$$

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