Mass Spectrum of Low lying Charmonium Spectrum in a Non relativistic Quark Model with Woods-Saxon Potential

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Introduction

Charmonium system is a powerful tool for the study of forces between quarks in QCD in non perturbative regime. Studies of charmonia production can improve our understanding of heavy quark production and the formation of bound states. Our work is based on a non-relativistic potential model framework which studies the mass spectra. The potential model works well in describing the heavy quarkonia states, especially for the charmonium states below the open-charm threshold. However, above this threshold, there are still many predicted states that have not been observed yet. In recent years, charmonium physics has gained renewed strong interest from both the theoretical and the experimental side, due to the observation of charmonium-like states, such as Y(4260), Y(4360), and Y(4660)[2]. A narrow resonance X(3872) was first observed in exclusive B decays $B^{\pm} \rightarrow K^{\pm}X(3872)$, $X(3872) \rightarrow \pi^+\pi^- J/\psi$ by Belle collaboration, with mass $M = 3872.0 \pm 0.6(\text{stat}) \pm 0.5(\text{sys})$ MeV. The CDF II collaboration, D0 collaboration and BaBar collaboration. $Y(3940)$ was discovered in exclusive B decays $B \rightarrow KY(3940)$, $Y(3940) \rightarrow \omega J/\psi$ by Belle collaboration. $X(3940)$ was observed by Belle collaboration in $e^+e^- \rightarrow J/\psi X(3940)$, $X(3940) \rightarrow D^*\bar{D}$ with mass $M = 3943 \pm 6 \pm 26$ MeV. $Y(4260)$ was first observed by BaBar collaboration in initial state radiation events $e^+e^- \rightarrow \gamma_{\text{ISR}}Y(4260)$, $Y(4260) \rightarrow \pi^+\pi^- J/\psi$ with mass $M \sim 4260$ MeV. This state was confirmed by CLEO collaboration. Other decay channels such as $Y(4260) \rightarrow \pi^0\pi^0 J/\psi$ and $Y(4260) \rightarrow K^+K^- J/\psi$ have also been observed by CLEO collaboration. Many other experimentally observed exotic states such as X(3915), Z(3930), etc have lead to the deeper understanding of the charmonium physics and unravelled many mysteries[1]

Theoretical Background

In a potential model approach the entire dynamics of quarks in a meson is governed by a Hamiltonian has kinetic energy term (K) and a potential energy (V), that is [3],

$$H = K + V.$$
$$K = M + \frac{p^2}{2\mu}$$

Here p is the relative momentum, $\mu = \frac{m_Q m_{\bar{Q}}}{m_Q + m_{\bar{Q}}}$ is the reduced mass of the $Q\bar{Q}$ system, where m_Q and $m_{\bar{Q}}$ are the masses of the individual quark and anti quark respectively and M is the total mass of quark and antiquark [3].
In most of the cases quark-antiquark potentials ($V(r)$) are reasonably considered inorder to correlate results with experiment. Quark-antiquark potential may be of the form;

$$V(r) = V_{ws} + V_{CONF}(r) + V_0 \tag{1}$$

the woods saxon potential V_{ws} defined as,

$$V_{WS} = \frac{-v_0}{1 - exp(\frac{r-R}{a})} \tag{2}$$

where r is the radial separation between the two heavy quarks and α_s is the strong coupling constant and V_0 is constant. For our model we have chosen the linear potential which represents the non perturbative effect of QCD that confines quarks within the color singlet system [4].

$$V_{CONF}(r_{ij}) = -a_c r_{ij} \lambda_i \cdot \lambda_j \tag{3}$$

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where a_c is the confinement strength and λ_i and λ_j are the generators of the color SU(3) group for the i^{th} and j^{th} quarks. It should be noted that the two body confinement potential, has symmetric and antisymmetric terms. In order to obtain the $Q\bar{Q}$ spectrum, we have solved the Shrodinger equation with Hamiltonian equation using the variational method:

$$E(\psi) = \frac{\langle\psi|H|\psi\rangle}{\langle\psi|\psi\rangle} = \langle H\rangle$$

In our work, we have used the three-dimensional harmonic oscillator wave function which has been extensively used in atomic and nuclear physics is used as the trial wave function for obtaining the $Q\bar{Q}$ mass spectrum.

$$\psi_{nlm}(r,\theta,\phi) = N\left(\frac{r}{b}\right)^l L_n^{l+1/2}(\frac{r}{b})exp\left(-\frac{r^2}{2b^2}\right)Y_{lm}(\theta,\phi)$$

where $|N|$ is the normalizing constant given by

$$|N|^2 = \frac{2\alpha^3 n!}{\sqrt{\pi}}\frac{2^{(2(n+l)+1)}}{(2n+2l+1)!}(n+l)! \qquad (4)$$

and $L_n^{l+1/2}(x)$ are the associated Laguerre polynomials,

Results and Discussions

There are five parameters in our model. These are the the mass of the bottom quark M_b, the confinement strength a_c, the quark-gluon coupling constant α_s, potential depth V_0, R and a [7].These parameters are adjusted until the energy of the trial wavefunction is minimized. The resulting trial wavefunction and its corresponding energy are variational method approximations to the exact wavefunction and energy. Within the framework of the non-relativistic potential model, the trial wave function of the heavy quarkonium satisfies the Schrodinger equation. The variational method has extensively been used and seems to be successful in many aspects-.

$V_0 \approx 50$ MeV; $a \approx 0.6$ fm; $R = 8$ fm; $M_b = 4.645$ GeV; $a_c = 135.0$ MeV fm^{-1}

Table 1 The Mass Spectrum of Charmonim (MeV)

$n^{2S+1}L_J$	The Mass	M_{exp} MeV	[2] MeV
1^1S_0	2982.53	2983.60±0.7	2990.40
2^1S_0	3635.37	3639.2±0.11	3646.50
3^1S_0	3963.60	....	4071.90
4^1S_0	4353.60	....	....
5^1S_0	4586.42	....	....
1^3S_1	3094.54	3096.916±0.011	3085.10
2^3S_1	3684.38	3686.108±0.018	3682.10
3^3S_1	4040.62	4039±1	4100.20
4^3S_1	4453.00	....	....
5^3S_1	4473.29	....	....

even though various forms of $q\bar{q}$ potentials have een predicted, potential model approach is still a powerful tool for investigating the properties of quarkonia owing to its great predictive power and mathematical simplicity. Also, the potential model provides a framework to understand the structure of the spin dependent interaction in QCD,the form of confinement,Coupled channel effects decay properties etc.

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