Effects of Coupled Channels in Charmonium Masses with Instanton Potetnial

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Introduction

The bound states of a charm and an anti-charm quark($c\bar{c}$) have enabled a revolution in our understanding of the heavy quarkonium spectroscopy which represents an important testing ground for the properties of the strong interaction. Charmonium system is a powerful tool for the study of forces between quarks in QCD in non perturbative regime. Studies of charmonia production can improve our understanding of heavy quark production and the formation of bound states. Our work is based on a non-relativistic potential model framework which studies the mass spectra of $c\bar{c}$ states taking into consideration the coupled channel effects. The major deficiency of any purely phenomenological model is that it includes only $q\bar{q}$ components of the Fock space expansion and totally neglects higher Fock space components which can be included as coupled channel effects. These are expected to be the most prominent for states close to threshold. The reason for some of the drawbacks of potential models is that the ignorance of coupled channel effects. As the mass of a quarkonium state approaches the threshold for decay to pairs of flavoured mesons, contributions from virtual loops of the flavoured meson channels are expected to make important contributions to masses and other meson properties. The masses obtained in the naive quark models are shifted by the inclusion of these coupled channel effects. We adopt the widely used $^3P_0$ pair creation model to include the coupled channel effects [3].

Theoretical Background

In a potential model approach the entire dynamics of quarks in a meson is governed by a Hamiltonian has kinetic energy term (K) and a potential energy (V), that is, [1]

$$H = K + V.$$

The potential energy V is the sum of the heavy-quark potential $V_{Q\bar{Q}}(\vec{r})$,linear confining potential $V_{conf}(\vec{r})$ and Coulomb potential $V_{coul}(\vec{r})$.

$$V_{Q\bar{Q}}(\vec{r}) = V_C(\vec{r}) + V_{SD}(\vec{r}).$$

Here $V_C(\vec{r})$ and $V_{SD}(\vec{r})$ are central and spin dependent potentials due to instanton vacuum respectively [2].

$$V_C(\vec{r}) \simeq \frac{4\pi\bar{\rho}^3}{\bar{R}^4 N_c}\left(1.345\frac{r^2}{\bar{\rho}^2} - 0.501\frac{r^4}{\bar{\rho}^4}\right) \quad (1)$$

Here, $\bar{\rho} = \frac{1}{3}$ fm the average size of the instanton, $\bar{R} = 1$ fm the average separation between instantons and number of colors N_C is 3.

$$V_{SD}(\vec{r}) = V_{SS}(\vec{r}) + V_{LS}(\vec{r}) + V_T(\vec{r})$$

$$V_{SS}(\vec{r}) = \frac{1}{3m_Q{}^2}\nabla^2 V_C(\vec{r});\ \ V_{LS}(\vec{r}) = \frac{1}{2m_Q{}^2}\frac{1}{r}\frac{dV_C(\vec{r})}{dr};$$

$$V_T(\vec{r}) = \frac{1}{3m_Q{}^2}\left(\frac{1}{r}\frac{dV_C(\vec{r})}{dr} - \frac{d^2V_C(\vec{r})}{dr^2}\right).$$

In our work, we have used the three-dimensional harmonic oscillator wave function.

Coupled Channel Effects

In this model it is assumed that the strong decay of meson A (with a quark q_1 and an anti-quark q_2) takes place through the creation of a pair of quark(q_3) and anti-quark(q_4) from vacuum with quantum number $J^{PC} = 0^{++}$. The created quark anti-qurak pair recombines with the quark and anti-qurak in the initial meson state forming final meson states i.e, mesons B and C. In the coupled channel model, the full hadronic state is given by [3]

$$|\psi\rangle = |A\rangle + \sum_{BC}|BC\rangle$$

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for open flavour strong decay $A \to BC$. Here A, B, C denote mesons. The wave function $|\psi\rangle$ obeys the equation

$$H|\psi\rangle = M|\psi\rangle \quad (2)$$

The Hamiltonian H for this combined system consists of a valence Hamiltonian H_0 and an interaction Hamiltonian H_I which couples the valence and continuum sectors. The matrix element of the valence-continuum coupling Hamiltonian is given by.

$$\langle BC|H_I|A\rangle = h_{fi}\delta(\vec{P}_A - \vec{P}_B - \vec{P}_C) \quad (3)$$

where h_{fi} is the decay amplitude.
The mass shift of meson A due to its continuum coupling to BC can be expressed in terms of partial wave amplitude $\mathscr{M}_{LS}$.

$$\Delta M_A^{(BC)} = \int_0^\infty dp \frac{p^2}{E_B + E_C - M_A - i\varepsilon} \int d\Omega_p |h_{fi}(p)|^2 = \int_0^\infty dp \frac{p^2}{E_B + E_C - M_A - i\varepsilon} \sum_{LS} |\mathscr{M}_{LS}|^2$$

$$\Delta M_A^{(BC)} = \mathscr{P} \int_0^\infty dp \frac{p^2}{E_B + E_C - M_A} \sum_{LS} |\mathscr{M}_{LS}|^2 + i\pi \left(\frac{p * E_B * E_C}{M_A} \sum_{LS} |\mathscr{M}_{LS}|^2 \right) |_{E_B+E_C=M_A}$$

Results and Discussions

The spectrum of charmonium states below threshold is calculated taking into account coupling to the pairs of lowest D and D_s mesons. The analysis is performed within the $^3P_0$ model for light -quark pair creation. To calculate mass shift we need to know the physical mass M of the charmonium using a non relativistic quark model using an instanton potential as shown in table 1. Following the instanton model we constructed the model pair-creation interaction from the potential in the quark model. It includes linear confinement and Coulomb like one gluon exchange potential. We evaluate the bare state masses and shifts due to DD,DD^*,D^*D^* D_sD_s,$D_sD_s^*$ and $D_s^*D_s^*$, loops (with $M_D = 1869.61$ MeV, $M_{D_s} = 1968.30$ MeV, $M_{D^*} = 2006.96$ MeV and $M_{D_s^*} = 2112.1$ MeV). The mass shifts calculated in our model due to the hadron loop effects are listed in Table 2.

Table 1 The Mass spectrum of charmonium (MeV)

$n^{2S+1}L_J$	Name	The Mass MeV	M_{exp} MeV	M_I MeV
1^1S_0	$\eta_c(1S)$	2984	2983.6±0.7	16.39
1^3S_1	J/ψ	3097	3096.916±0.011	14.92
2^1S_0	$\eta_c(2S)$	3640	3639.2±0.11	17.91
2^3S_1	$\psi(2S)$	3687	3686.108±0.018	17.84
1^3P_0	$\chi_{c0}(1P)$	3414	3414.75±0.31	7.41
1^3P_1	$\chi_{c1}(1P)$	3510	3510.66±0.07	6.91
1^1P_1	$h_c(1P)$	3525	3525.38±0.11	6.14
1^3P_2	$\chi_{c2}(1P)$	3555	3556.20±0.09	6.77
2^3P_0	$\chi_{c0}(2P)$	3916	3915±3±2	12.09
2^3P_1	$\chi_{c1}(2P)$	3872	3872	12.54
2^1P_1	$h_c(2P)$	3927	....	11.84
2^3P_2	$\chi_{c2}(2P)$	3929	3927.2±2.6	12.24
1^3D_3	$\psi_2(1D)$	3845	....	5.33
1^1D_2	$\eta_{c2}(1D)$	3812	...	4.62
1^3D_2	$\psi_2(1D)$	3823	3823	5.32
1^3D_1	$\psi_1(1D)$	3779	3778	5.42

Table 2 The mass shifts of charmonium states in MeV

$n^{2S+1}L_J$	DD	DD^*	D^*D^*	D_sD_s	$D_sD_s^*$	$D_s^*D_s^*$	Total
1^3S_1	14	54	72	13	31	54	238
1^1S_0	0	64	53	0	43	48	208
1^3P_2	16	41	86	12	22	41	218
1^3P_1	0	64	68	0	28	31	191
1^3P_0	28	0	64	10	0	39	141
1^3P_1	0	61	67	0	25	36	189
2^3S_1	12	22	34	5	14	12	99
2^1S_0	0	31	24	0	15	14	84
2^3P_2	6	18	22	4	6	8	64
2^3P_1	0	24	15	0	8	11	58
2^3P_0	12	0	15	3	0	8	38
2^1P_1	0	22	14	0	10	5	51
1^3D_3	12	22	58	6	10	8	116
1^3D_2	0	52	38	0	12	19	121
1^3D_1	33	22	41	5	6	18	125
1^1D_2	0	58	33	0	12	9	112

References

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