Properties of Bottomonium with Coupled Channel Effects

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Introduction

The properties of bottomonium is studied in relativistic limit using relativistic harmonic model. In addition coupled channel effect for $n = 1$ state of bottomonium has been encorporated in this work. Virtual hadron loops will induce mass shifts in the spectrum of mesons. We have considered formalism of coupled channel effect in Bottomonium under Quark Pair Creation (QPC) model. According to it quark pairs are created with vacuum quantum numbers. Because of this assumption in QPC model, it is also called as $^3P_0$ model. Hadron loop effects are studied extensively for charmonium, there is a good scope for further study in bottomonium sector as well[1][3][4].

Formalism

The hadron loop effects leads to two body strong decays of the meson above threshold and below threshold they give rise to mass shifts of the bare meson states and it is well formluated using $^3P_0$ model[2][5].

By considering $|A\rangle$ as bare state of bottomonium and continuum coupling with mesonic loops, $A \to BC$, the full hadronic loop is described as,

$$|\psi\rangle = |A\rangle + \sum_{BC} |BC\rangle \tag{1}$$

this is corresponding for open flaovour strong decay . Here A, B and C denote mesons.

The wave function $|\psi\rangle$ obeys the eigen value - equation

$$H|\psi\rangle = M|\psi\rangle \tag{2}$$

The Hamiltonian H for this combined system consists of a valence Hamiltonian H_0 and an interaction Hamiltonian H_I which couples the valence and continuum sectors.

$$H = H_0 + H_I \tag{3}$$

where

$$H_I = g \int d^3x \bar{\psi}\psi \tag{4}$$

The matrix element of the valence-continuum coupling Hamiltonian is given by [**?** **?**]

$$\langle BC|H_I|A\rangle = h_{fi}\delta(\vec{P}_A - \vec{P}_B - \vec{P}_C) \tag{5}$$

where h_{fi} is the decay amplitude.

The mass shift of meson A due to its continuum coupling to BC can be expressed in terms of partial wave amplitude $\mathscr{M}_{LS}$ as

$$\Delta M_A^{(BC)} = \int_0^\infty dp \frac{p^2}{E_B + E_C - M_A - i\varepsilon} \int d\Omega_p |h_{fi}(p)|^2$$
$$= \int_0^\infty dp \frac{p^2}{E_B + E_C - M_A - i\varepsilon} \sum_{LS} |\mathscr{M}_{LS}|^2$$

$$\Delta M_A^{(BC)} = \mathscr{P} \int_0^\infty dp \frac{p^2}{E_B + E_C - M_A} \sum_{LS} |\mathscr{M}_{LS}|^2$$
$$+ i\pi \left(\frac{p * E_B * E_C}{M_A} \sum_{LS} |\mathscr{M}_{LS}|^2 \right) |_{E_B + E_C = M_A}$$

The decay amplitude is given by;

$$\frac{d\Gamma_{A\to BC}}{d\Omega} = 2\pi P \frac{E_B E_C}{M_A} |h_{fi}|^2 \tag{6}$$

where in the rest frame of A, we have $\vec{P}_A = 0$ and $P = |\vec{P}_B| = |\vec{P}_C|$.

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$$P = \sqrt{[M_A^2 - (M_B + M_C)^2][M_A^2 - (M_B - M_C)^2]}/(2M_A) \quad (7)$$

and the total decay rate is given by:

$$\Gamma_{A\to BC} = 2\pi P \frac{E_B E_C}{M_A} |\mathscr{M}_{LS}|^2 \quad (8)$$

Results and Discussion

The Hamiltonian matrix is constructed for evaluation of mass spectrum of bottomonium in the harmonic oscillator basis. Oscillator wave function is of the form;

$$\psi_{nlm}(r,\theta,\phi) = N\left(\frac{r}{b}\right)^l L_n^{l+1/2}(\frac{r}{b}) exp\left(-\frac{r^2}{2b^2}\right) Y_{lm}(\theta,\phi)$$

where N is the normalising constant given by

$$|N|^2 = \frac{2n!}{b^3\pi^{1/2}} \frac{2^{[2(n+l)+1]}}{(2n+2l+1)!}(n+l)! \quad (9)$$

$L_n^{l+\frac{1}{2}}$ are the associated Laguerre polynomials.

The meson continuum loops for bottomonium comprises of $B\bar{B}$, $B\bar{B}^*$, $B^*\bar{B}^*$, $B_s\bar{B}_s$, $B_s\bar{B}_s^*$ and $B_s^*\bar{B}_s^*$ virtual loops.

In the calculation of coupled channel effect in bottomonium, following parameters are used;

$M_B = M_{\bar{B}} = 5279.26$ MeV, $M_B^* = M_{\bar{B}^*} = 5324.6$ MeV, $M_{B_S} = M_{\bar{B}_S} = 5366.77$ MeV and $M_{B_S^*} = M_{\bar{B}_S^*} = 5279.26$ MeV,

TABLE I: The mass shifts due to coupled channel effect for $n = 1$ state of bottomonium are shown here, δM depicts total mass shift

Bare States	Mass (M)	$B\bar{B}$	$B\bar{B}^*$	$B^*\bar{B}^*$	$B_s\bar{B}_s$	$B_s\bar{B}_s^*$	$B_s^*\bar{B}_s^*$	δM
$1\,^1S_0$	9399.74	-	15.9208	30.7863	-	25.0072	14.0296	87.9156
$1\,^3S_1$	9460.77	5.7741	11.1328	21.5924	5.0472	9.7534	18.957	72.257
$1\,^1P_1$	9898.95	-	12.3364	23.3453	-	10.0834	19.211	64.9761
$1\,^3P_0$	9856.75	11.0019	-	51.7859	9.1102	-	43.2673	115.1653
$1\,^3P_1$	9903.73	-	25.8125	49.3087	-	21.488	41.3902	137.9994
$1\,^3P_2$	9919.21	11.7763	16.7083	59.0753	9.6476	13.7804	49.0186	160.0062
$1\,^1D_2$	10160.69	-	16.7251	60.8227	-	12.6966	48.4866	138.731
$1\,^3D_1$	10151.14	6.9886	3.3010	82.4288	5.3091	2.5574	65.463	166.048
$1\,^3D_2$	10163.48	-	17.0485	42.9376	-	12.6208	33.386	105.9929
$1\,^3D_3$	10170.97	16.1279	19.9289	13.0108	12.3683	15.4923	9.7946	86.7228

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