Magnetic moments of decuplet baryons using effective quark masses in statistical model

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Introduction

The magnetic moments of baryons have been theoretically investigated using different approaches, such as: simple additive quark model in non-relativistic limit which calculates the magnetic moments of the baryons as the sum of its constituent quark magnetic moments. The subject of magnetic moments is complicated to explain or understand because the phenomenon of baryons is contributed from the valence quarks magnetic moments as well as from other effects like contributions from pion cloud, relativistic effects, etc. Many theoretical formalisms have been successful to calculate the magnetic moments of $J^P = 3/2^+$ decuplet baryons. The relativistic quark model (RQM) [1], QCD-based quark model (QCDQM) [2], effective mass scheme (EMS) [3], light cone QCD sum rule (LCQSR) [4] etc. are few to name. The results are compared with the predictions of other models.

Theoretical Framework

The model is based on assumption of hadrons as an ensemble of quark-gluon Fock states such that each Fock state shares some part of total probability associated with quark-gluon Fock states. The Fock states include different sub-processes like etc. $q \Leftrightarrow qg, g \Leftrightarrow q\bar{q}, g \Leftrightarrow gg$
The methodology is based on framing a suitable wave-function such that each term in the wave-function contains suitable combinations of valence and sea quarks such that it satisfies the anti-symmetrisation and spin 3/2, flavour decuplet, color singlet of baryonic system.

$$\left|\Phi^{\uparrow}_{\frac{3}{2}}\right\rangle = \frac{1}{N}[a_0\phi_1^{(\frac{3}{2})\uparrow} H_0 G_1 + b_1[\phi_1^{(\frac{3}{2})} \otimes H_1]^{\uparrow} G_1 + b_8[\phi_1^{(\frac{1}{2})} \otimes H_1]^{\uparrow} G_8 + d_1[\phi_1^{(\frac{3}{2})} \otimes H_2]^{\uparrow} G_1 + d_8[\phi_8^{(\frac{1}{2})} \otimes H_2]^{\uparrow} G_8]$$

$$where N^2 = a_0^2 + b_1^2 + b_8^2 + d_1^2 + d_8^2$$

a_0 term describe a spin 3/2 of q^3 coupled to a spin 0 (scalar sea) , b_1:b_8 describes vector sea and d_1;d_8 signifies tensor sea. To calculate the coefficients, we use principle of detailed balance [5] in order to find the flavor probability of each Fock state. The principle assumes that every Fock state should be balanced by other Fock states.
The expressions for the rates of transitions between any two processes can be written as:

1. When both the $q \Leftrightarrow qg, g \Leftrightarrow gg$ processes are included:

$$\frac{\rho_{i,j,l,k}}{\rho_{i,j,l,k-1}} = \frac{(3+2i+2j+2l+k-1)}{(3+2i+2j+2l)k + \frac{k(k-1)}{2}}$$ where i

refer to $u\bar{u}$ pairs, j refer to $d\bar{d}$, l refers to $s\bar{s}$ and k refers to no. of gluons.

2. When the processes $g \Leftrightarrow s\bar{s}$ are included: The exchanges between the gluons and strange quark anti-quark pair are limited due to large mass of strange quark. Gluon must possess the free energy at least greater than mass of strange quark. Using free energy distributions for gluons, the expression for transition rates between gluon and strange quark anti-quark pair can be written as:

$$\frac{\rho_{i,j,l,k}}{\rho_{i,j,l+k,0}} = \frac{k(k-1)...1(1-C_0)^{n-2l-1}...(1-C_{l-1})^{n-k-2}}{(l+1)(l+2)....(l+k)(l+k+1)}$$

The statistical method involves the computation of individual multiplicities denoted in the form of various ratios.

$$\rho_{\frac{1}{2}1}[\rho_{88}, \rho_{10\overline{10}}] = c[2,1] = d[\frac{1}{96}, \frac{1}{300}]$$

$$\rho_{\frac{1}{2}2}[\rho_{11}, \rho_{88}] = 2c[1,2] = 2d[\frac{1}{5}, \frac{1}{160}]$$

The product probability ratios are expressed in the form of common parameter “c” and “d”. These coefficients provide us the contribution from sea. Detailed calculations are shown in reference 5.

Magnetic Moments of $J^P = 3/2^+$ baryons

Magnetic moments is low energy and long distance phenomenon. The magnetic moments of decuplet baryons are computed using the spin-flavor wave functions of the constituting quarks. Here, we have incorporated the effect of quark effective masses to compute the magnetic moments. The quark magnetic moment moments in terms of effective quark masses are shown in reference 6. The magnetic moment operator over a baryonic wave function can be defined as:

$$
\begin{aligned}
\left\langle \Phi^{\uparrow}_{\frac{3}{2}} \middle| \hat{O} \middle| \Phi^{\uparrow}_{\frac{3}{2}} \right\rangle &= \frac{1}{N}[\left\langle \phi_1^{(\frac{3}{2})\uparrow} H_0 G_1 \middle| \hat{O} \middle| \phi_1^{(\frac{3}{2})\uparrow} H_0 G_1 \right\rangle + \\
&\left\langle [\phi_1^{(\frac{3}{2})} \otimes H_1]^{\uparrow} G_1 \middle| \hat{O} \middle| [\phi_1^{(\frac{3}{2})} \otimes H_1]^{\uparrow} G_1 \right\rangle + \\
&\left\langle [\phi_1^{(\frac{1}{2})} \otimes H_1]^{\uparrow} G_8 \middle| \hat{O} \middle| [\phi_1^{(\frac{1}{2})} \otimes H_1]^{\uparrow} G_8 \right\rangle + \\
&\left\langle [\phi_1^{(\frac{3}{2})} \otimes H_2]^{\uparrow} G_1 \middle| \hat{O} \middle| [\phi_1^{(\frac{3}{2})} \otimes H_2]^{\uparrow} G_1 \right\rangle + \\
&\left\langle [\phi_8^{(\frac{1}{2})} \otimes H_2]^{\uparrow} G_8 \middle| \hat{O} \middle| [\phi_8^{(\frac{1}{2})} \otimes H_2]^{\uparrow} G_8 \right\rangle
\end{aligned}
$$

For Ξ^{*-} , the expression for the magnetic moments can be written as:

$$
\begin{aligned}
\Phi^{\uparrow}_{\frac{3}{2}} &= \frac{1}{N^2}[a_0^2(30\mu_s + 15\mu_d) + \\
&b_1^2(22\mu_s + \mu_d) + b_8^2(22\mu_s + \mu_d) + \\
&d_1^2(6\mu_s + 3\mu_d) + d_8^2(3\mu_s + 1.5\mu_d)]
\end{aligned}
$$

The quark masses used in above relation are quark effective masses. We repeat our computations by varying the effective masses of quarks (in MeV) from 360 to 390 for u and d quarks and 500 to 540 for strange quark, to have an idea about the most suitable set of effective quark masses that yields the quark magnetic moments and hence magnetic moments of baryons. As the values of effective masses are model dependant so the magnetic moments of quarks are also model dependent and we have to take their values compatible with the constituent quark masses. The results of our work are compared with the predictions of other models in the table shown below:

Particle	Magnetic moments	Experimental Results[7]
Δ^{++}	5.50	4.52±0.50
Δ^{+}	3.1	2.7±1.5± 3
Δ^{0}	-0.08	-
Δ^{-}	-2.89	-
$\sum^{*+}$	3.05	-
$\sum^{*0}$	0.28	-
$\sum^{*-}$	-2.56	-
Ξ^{*0}	0.63	-
Ξ^{*-}	-2.17	-
Ω^{-}	-1.83	-2.02±0.05

Results and Conclusions

In the present paper, the baryon decuplet wave function is studied, by including the concept of effective mass to estimate the magnetic moments. Effective quark masses for quarks u, d, and s are calculated using fixed inputs for baryon masses (PDG) and statistical parameters as input in the respective formulae. These effective masses of u, d and s are acting as an input to the magnetic moments of baryon decuplet. Our results give a good match with the experimental values as seen in above table..

References

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