Multifractal Analysis of Multiplicity Fluctuations in ^{32}S - AgBr Interactions at 200 AGeV

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Introduction

The study of non-statistical fluctuations in relativistic nuclear collisions has recently attracted a great deal of attention due to the possibility of extracting important information about the mechanism of multiparticle production in such collisions. To understand the multifractal structure of multiplicity distributions of produced pions in pseudorapidity phase space, Takagi [1,2] proposed a new method known as Takagi moment. In this method, a single event contains n relativistic singly charged particles distributed in the interval $\eta_{min} < \Delta\eta < \eta_{max}$ in η-phase space. The multiplicity of *n* shower particles changes from event to event according to the distribution $Pn(\Delta\eta)$ where, $\Delta\eta = \eta_{min} - \eta_{max}$. The pseudo-rapidity interval of single charged particles is divided into M bins of equal size $\delta\eta = \Delta\eta/M$. The multiplicity distribution for a single bin is denoted as $P_n(\delta\eta)$ for n = 0, 1, 2, 3,-----, where it is assumed that the inclusive rapidity distribution $dn/d\eta$ is constant and $P_n(\delta\eta)$ is independent of the location of the bin. If Ω denotes the number of independent events, then the shower particles (mostly pions) produced in Ω events are distributed in ΩM bins of size $\delta\eta$. Let N be the total number of hadrons produced in these events and n_{ij} the multiplicity of pions in the j^{th} bin of the i^{th} event. Thus according to the theory of multifractals [3], the normalized density, P_{ij}, is defined as:

$$P_{ij} = n_{ij}/N \tag{1}$$

This is also true for $N \to \infty$. The Takagi moment of order *q* is defined as:

$$T_q(\delta\eta) = \ln \sum_{i=1}^{\Omega}\sum_{j=1}^{M} P_{ij}^q \quad \text{for } q > 0 \tag{2}$$

It is expected to behave like a linear function of the logarithm of the “resolution” $R(\delta\eta)$

$$T_q(\delta\eta) = A_q + B_q \ln R(\delta\eta) \tag{3}$$

where, A_q and B_q are constants independent of $\delta\eta$. If such a linear relation is observed for a considerable range of $R(\delta\eta)$, a generalized dimension may be defined as:

$$D_q = B_q/(q-1) \tag{4}$$

For a sufficiently large number of events Ω the summation in Eqn. (2) is evaluated as:

$$\sum_{i=1}^{\Omega}\sum_{j=1}^{M} P_{ij}^q = \langle n^q \rangle/(N^{q-1}\langle n \rangle) \tag{5}$$

Using Eqns. (2-5), the following linear relation is easily obtained for the simplest choice $R(\delta\eta)=\delta\eta$.

$$\ln \langle n^q \rangle = A_q + \{(q-1)D_q + 1\}\ln\delta\eta \tag{6}$$

From Eqn. (6), one gets the following relation:

$$\ln \langle n^q \rangle = A_q + \{(q-1)D_q + 1\}\ln\langle n \rangle$$
$$= A_q + K_q \ln\langle n \rangle \tag{7}$$

where, the slope K_q is related to the generalized dimension, D_q , by the following relation:

$$D_q = (K_q - 1)/(q-1) \tag{8}$$

The existence of linear behaviour over a considerable range of <*n*> indicates the presence of the fractal structure in the multiparticle production. The values of D_q for q ≥ 2 in expression (4) can be determined easily from the slopes of the plot between $\ln \langle n^q \rangle$ and $\ln \langle n \rangle$. The value of information dimension, D_1, for q = 1 can be also be determined from a new and simple relation suggested by Takagi

$$\langle n\ln n\rangle/\langle n \rangle = C_1 + D_1 \ln\langle n \rangle \tag{9}$$

where C_1 is a constant.

Experimental Technique

In this experiment two stacks of Ilford G5 nuclear emulsion plates exposed horizontally to a ^{32}S-beam at 200 AGeV obtained from Super Proton Synchrotron (SPS) at CERN have been utilized for data collection. The other relevant details about the present experiment can be found in our earlier publication [4-6].

Results and Discussions

In order to investigate the multifractality of relativistic shower particles produced in ^{32}S - AgBr interactions at 200 AGeV in η – space, the analysis of the multiplicity distribution in the central pseudo-rapidity region $\eta_{peak} - 1.5 < \eta_{peak} + 1.5$ has been considered, which covers most of the produced shower particles. The values of $<n ln n>/n$ and $<n^q>$ are computed for each interval and are shown in Fig. 1. The values of $ln<n^q>$ as a function of $ln< n >$ for different q are shown in figure for ^{32}S – AgBr collisions at 200 AGeV. All the experimental points clearly follow an excellent linear relation in each plot for the whole range of $<n>$. The errors shown in the plots are purely statistical. The values of the slopes, K_q, were obtained by fitting eqn. (7) to the experimental points. The linear behaviour of $ln<n^q>$ with $ln< n >$ in the figure gives an indication of fractal structure in multiparticle production in η-space.

The experimental data has been compared with the FRITIOF data and are also plotted in Fig. 1. The observed points follow an excellent linear relation for the whole range of $< n >$ similar to the experimental data. Hence, it has been found that the experimental data on multifractality exhibit a remarkable closeness to analogous data obtained from the FRITIOF model.

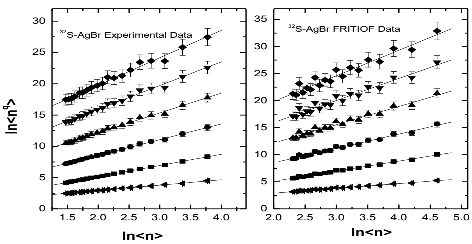

Fig. 1: Variation of $ln<n^q>$ with $ln<n>$.

The values of information dimension (D_1) obtained using eqn. (9) for ^{32}S – AgBr collisions and FRITIOF data are found to be 0.871 ± 0.007 and 0.870 ± 0.008 respectively and are almost same. The values of the generalized dimensions, D_2, D_3, D_4, D_5 and D_6 are computed from the best fitted slopes of linear relation of $ln<n^q>$ with $ln< n >$ for both experimental and FRITIOF data. The values of generalized dimensions, D_q are shown in Fig. 2 for the experimental and FRITIOF data It is noticed from the figure that the values of generalized dimensions, D_q decreases with increasing order of moment, q, thereby showing multifractality in pion production for the interaction, which in turn supports an interpretation in terms of a cascade mechanism in the multiparticle production process in pseudo-rapidity phase space. So the present analysis of the multifractal structure shows a remarkable property of the observed fluctuations.

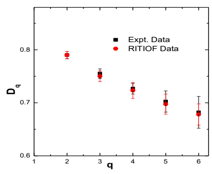

Fig. 2: Variation of Generalized fractal dimension D_q with order of moments q.

Further, the values of specific heat can be found from the slope of best linear fits to the D_q against $lnq/(q-1)$ as shown in Fig.3.Values of Multifractal specific heats of our data along with the FRITIOF data are found to be 0.28 ± 0.01 and 0.31 ± 0.01 respectively. The linear behaviour of D_q with $lnq/(q-1)$ shows reasonably good agreement between the experimental data and the multifractal Bernoulli representation and the multifractal specific heat seems to have a universal value (~ ¼) that does not depend on projectile and their energy.

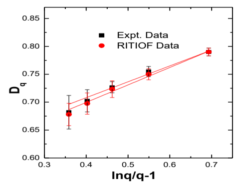

Fig. 3: Plot of D_q with $lnq/(q-1)$.

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