Magnetic and dynamical properties of QCD medium in a strong magnetic field

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Introduction

In recent years, the existence of a very strong but rapidly decaying magnetic field ($\sim 10^{18}$ Gauss) produced in noncentral ultrarelativistic heavy ion collision has generated an interest in QGP community. It is yet to be settled whether this strong magnetic field would last long enough for the medium to be formed. But, if the electrical conductivity of the medium is large enough, the transient magnetic field would induce a current which, in turn, would elongate the lifetime of the magnetic field consistent with the Lenz's law. Therefore it would be intriguing to study the effect of strong magnetic field on the deconfined medium and its subsequent expansion. Recently we have studied how the equation of state of thermal QCD matter with two light flavors is modified in the presence of a strong magnetic field by calculating the thermodynamic observables of hot QCD matter up to one-loop [1], where the pressure at fixed temperature increases with the magnetic field whereas the energy density is seen to decrease. As a result, the speed of sound of the QCD medium is increased due to the presence of strong magnetic field [1, 2], which might affect the hydrodynamic expansion. Recently the effect of magnetic field on the hydrodynamic equation of motion has also been studied in relativistic magnetohydrodynamics by calculating the energy-momentum tensor in an external magnetic field [3, 4]. Thus the aim of our article is twofold. The first one is to study the response of magnetic field on the hot QCD medium in the presence of a strong magnetic field, by calculating the first and second order cumulants of the free energy with respect to the magnetic field, *i.e.* magnetization and magnetic susceptibility. Both the magnetization and magnetic susceptibility are found to increase linearly with the magnetic field, thus it affirms the paramagnetic behavior of the hot QCD medium in a strong magnetic field.

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The second one is to explore the effect of strong magnetic field on the hydrodynamic expansion of the medium produced in the noncentral events either at RHIC or LHC, via the Bjorken boost-invariant expansion with the equation of state of hot QCD medium in the presence of strong magnetic field, where the effect of magnetic field as well as the initial magnetization have been manifested by the paramagnetic character, unlike the aforesaid works [3, 4], where the effect of magnetic field is encoded in the energy-momentum tensor.

Free energy and magnetization

The quark contribution to the free energy of a strongly magnetized hot QCD medium with two light flavors is calculated up to one-loop [1] as

$$\Omega_q = -6\int \frac{d^4p}{(2\pi)^4} \ln\left[\det\left(\gamma^{\parallel}\cdot p_{\parallel} - m_f - \Sigma(p_{\parallel})\right)\right] \quad (1)$$

and the gluon contribution consists of quark and gluon loops, where the former is obtained up to one-loop as

$$\Omega_g^{\rm quark-loop} = 8\int \frac{d^4p}{2(2\pi)^4} \ln\left[\mathbf{p}^2 + \Pi_{\parallel}(p_{\parallel})\right], \quad (2)$$

where $\Sigma(p_{\parallel})$ and $\Pi_{\parallel}(p_{\parallel})$ are the quark self-energy and the gluon self-energy due to quark-loop only in a strong magnetic field. However the form of gluon self-energy due to the gluon loops remains the same even in the presence of a strong magnetic field,

$$\Omega_g^{\rm gluon-loops} = 8\left[\int \frac{d^4p}{(2\pi)^4} \ln\left[p^2 + \Pi_T(p)\right] + \int \frac{d^4p}{2(2\pi)^4} \ln\left[\mathbf{p}^2 + \Pi_L(p)\right]\right]. \quad (3)$$

Here the Debye mass appeared in $\Pi_T(p)$ and $\Pi_L(p)$ is $\mathcal{O}(g'^2T^2)$ while that appeared in $\Pi_{\parallel}(p_{\parallel})$ is $\mathcal{O}(g^2|q_fB|)$. So the total Debye mass for two massless flavors depends on both the temperature and the magnetic field as [2],

$$m_D^2 = g'^2T^2 + \frac{g^2}{8\pi^2}\sum_f |q_fB|\,. \quad (4)$$

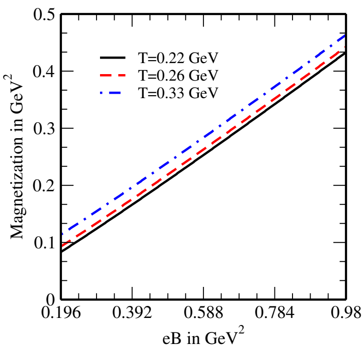

FIG. 1: Variation of magnetization with magnetic field at different temperatures.

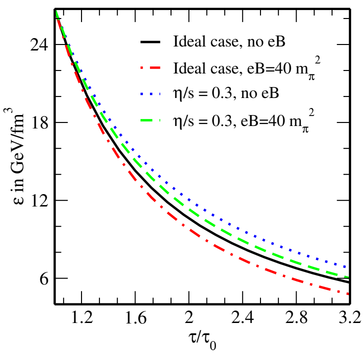

FIG. 2: Evolution of energy density in different cases.

We can now obtain the magnetization ($\mathcal{M}$) and magnetic susceptibility (χ) from the free energies (1-3),

$$\mathcal{M} = -\frac{\partial \Omega}{\partial (eB)} \ , \tag{5}$$

$$\chi = -\frac{\partial^2 \Omega}{\partial (eB)^2} \ , \tag{6}$$

where the magnetization is positive and increases with magnetic field (figure 1) and the magnetic susceptibility is also found to behave in the similar fashion [2] and hence it confirms the paramagnetic nature of the hot QCD medium in a strong magnetic field.

Bjorken expansion

The free energies (1-3) therefore yield the equation of state (EoS) ($p = c_s^2 \varepsilon$) for QCD medium in a strong magnetic field [1], where we have noticed the enhancement of the square of speed of sound (c_s^2) as compared to that in the absence of magnetic field and hence the hydrodynamic expansion could be affected. So we will now explore the Bjorken expansion of such a medium with the aforesaid EoS. The conservation of energy-momentum tensor for ideal fluid, $\partial_\mu T^{\mu\nu} = 0$ yields the hydrodynamic equation of motion, which, in conjunction with the EoS, gives the evolution of energy density with the proper time as

$$\varepsilon(\tau)\tau^{1+c_s^2} = \varepsilon(\tau_0)\tau_0^{1+c_s^2}, \tag{7}$$

with the given initial proper time (τ_0) and initial energy density ($\varepsilon(\tau_0)$). The effect of paramagnetic nature of EoS enters through the speed of sound. With the energy density of the form, $\varepsilon \propto T^4$, eq. (7) is translated into the cooling law for the ideal fluid: $T^3\tau = T_0^3\tau_0$ with T_0 as the initial temperature. However, in the presence of first-order dissipative forces, the conservation of the energy-momentum tensor gives

$$\frac{d\varepsilon}{d\tau} + \frac{\varepsilon + P}{\tau} = \frac{4s}{3\tau^2}\frac{\eta}{s} \ , \tag{8}$$

where η/s is the shear viscosity to entropy ratio. Similar to the ideal fluid, with the aforesaid EoS, the equation of motion (8) gives the evolution of energy density and temperature as functions of τ and η/s in a first-order dissipative hydrodynamics.

To see the effect of the strong magnetic field on the longitudinal expansion of paramagnetic QCD medium, we have plotted energy density as a function of the proper time for both ideal and dissipative hydrodynamic cases (figure 2), where we have set $\tau_0 = 0.3$ fm/c, $T_0 = 0.36$ GeV and through the equation of state the corresponding initial energy density is set at 26.74 GeV/fm^3. We have observed that, for both the aforesaid cases, the decrease of ε of QCD medium caused by the longitudinal expansion with the proper time is slightly faster in the presence of a strong magnetic field as compared to that in the absence of magnetic field which is an artefact of the increase of speed of sound in the strongly magnetized QCD medium.

References

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