Effect of causality on dissipation in relativistic fluid

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Introduction

The collision of heavy ions at ultra-relativistic energies create a deconfined state of matter, known as quark-gluon plasma (QGP). The evolution of this matter can be modelled by relativistic viscous hydrodynamics, such as Navier-Stoke's theory (1st order hydrodynamics). But the first order theory has a serious problem, it results in unstable solution and violates causality. These unphysical behaviors are cured by Muller, Israel and Stewart by introducing 2nd order theory, which gives stable solution and restores causality.

First order hydrodynamics takes transport coefficients such as shear viscosity, bulk viscosity, thermal conductivity etc. as inputs but in addition to these, the causal or 2nd order theory contains a few more thermodynamic functions, known as second-order coefficients such as different relaxation times and relaxation lengths for various dissipative fluxes. By setting these extra coefficients to zero in causal theory,one can get back the usual acausal first order theory.

In this work, we use 2nd order hydrodynamics to inspect the propagation of acoustic wave through dissipative fluid with non-zero net (baryonic) charge, shear viscosity, bulk viscosity and thermal conductivity and the effects of causality on the fluidity of QGP. We have used natural unit, *i.e.* $c = \hbar = k_B = 1$ here and the Minkowski metric is set as $g^{\lambda\mu} = diag.(-,+,+,+)$.

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1. Formalism

The 2nd order energy-momentum tensor is given by[1]:

$$T^{\lambda\mu} = \epsilon u^\lambda u^\mu + P\triangle^{\lambda\mu} + 2h^{(\lambda}u^{\mu)} + \tau^{\lambda\mu} \quad (1)$$

where the dissipation viscous stress tensor $\tau^{\lambda\mu} = \Pi\triangle^{\lambda\mu} + \pi^{\lambda\mu}$ with $\pi^\lambda_\lambda = h^\lambda u_\lambda = \tau^{\lambda\mu}u_\lambda = 0$. The projection operator is defined by $\triangle^{\lambda\mu} = g^{\lambda\mu} + u^\mu u^\lambda$ with $u^\mu u_\mu = -1$. The heat flux four vector is given by $q^\mu = h^\mu - n^\mu(\epsilon + P)/n$, the particle four flow $N^\mu = nu^\mu + n^\mu$ with $n^\mu u_\mu = 0$, where n is the net number density, Π is the bulk pressure, u^μ is the fluid four velocity, ϵ is the energy density, P is the thermodynamic pressure and $h(= \epsilon + P)$ is the enthalpy density. The symmetric tensor $h^{(\lambda}u^{\mu)}$ is defined as $h^{(\lambda}u^{\mu)} = \frac{1}{2}(h^\lambda u^\mu + h^\mu u^\lambda)$.

We use the Landau-Lifshitz(LL) frame, which represents a local rest frame for vanishing energy dissipation but non-zero net charge dissipation. In LL frame: $h^\mu = 0$, $n^\mu = -nq^\mu/(\epsilon + P)$ and the different viscous fluxes are given by [1]

$$\begin{aligned}
&\Pi = -\frac{1}{3}\zeta(u^\mu_{|\mu} + \beta_0 D\Pi - \alpha_0 q^\mu_{|\mu}) \\
&q^\lambda = \chi T\triangle^{\lambda\mu}[(\partial_\mu\alpha)nT/(\epsilon + P) - \beta_1 Dq_\mu \\
&+ \alpha_0\partial_\mu\Pi + \alpha_1\Pi^\nu_{\mu|\nu}] \\
&\Pi_{\lambda\mu} = -2\eta[u_{<\lambda|\mu>} + \beta_2 D\Pi_{\lambda\mu} - \alpha_1 q_{<\lambda|\mu>}]
\end{aligned} \quad (2)$$

where $D \equiv u^\mu\partial_\mu$, is co-moving derivative or material derivative. In the local rest frame, $D\Pi = \partial_0\Pi \equiv \dot{\Pi}$. The different coefficients appearing in Eq.2 are $\alpha = \mu/T$, is known as thermal potential, ζ is the coefficient of bulk viscosity, η is the coefficient of shear viscosity, χ is the coefficient of thermal conductivity, $\beta_0, \beta_1, \beta_2$ are relaxation coefficients, α_0 and

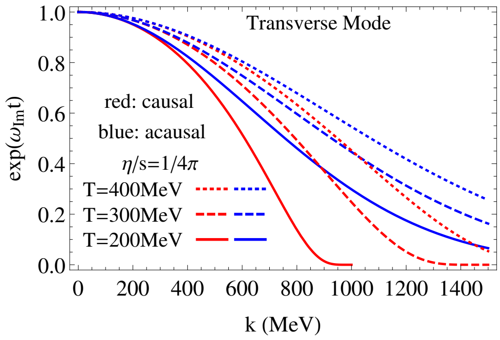

FIG. 1: Dissipation of perturbations with wave vector k at three different temperatures for vanishing bulk viscosity.

α_1 are coupling coefficients, are the signature of 2nd order theory. In order to investigate the role of causality on the dissipation of perturbation created in the fluid, we derive the dispersion relation relation (DR) using Eq.2. To do so, we decompose the fluid four velocity along and perpendicular to the wave vector (k), defined as longitudinal and transverse direction respectively. If we represent the perturbation by a wave then the imaginary part of the DR will damp it for increasing k, as shown in Fig.1 (transverse mode) and Fig.2 (longitudinal mode). It is clear from the results that, causality enhances the dissipation for fluid with respect to the acausal case due the inclusion of different relaxation times present in the causal theory. We use this DR to investigate the fluidity measure of the fluid, which is defined through the real and imaginary part of the DR. Fig.3 shows the variation of the fluidity measure (F) with temperature for non zero shear viscosity and vanishing bulk viscosity. It clearly shows that, causality increases F, that means, the viscosity of fluid enhances and that results the fluid become less perfect. The discontinuity at the transition temperature T_c originates from the first order phase transition, which is assumed here.

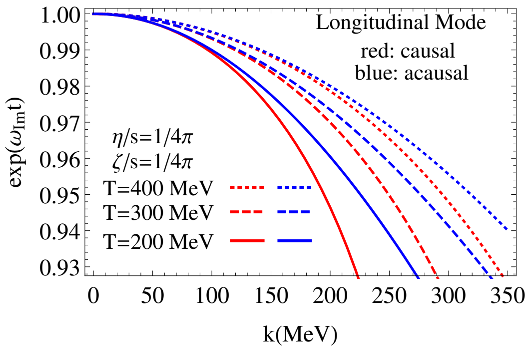

FIG. 2: Dissipation of perturbations with wave vector k at three different temperatures for non-vanishing bulk viscosity.

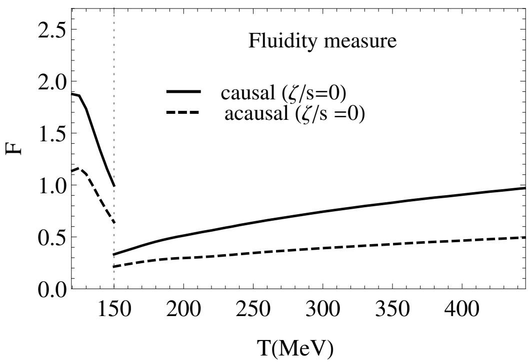

FIG. 3: Temperature variation of fluidity measure for vanishing bulk viscosity.

Acknowledgments

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References

[1] W. Israel and J. M. Stewart, Ann. Phys. **118**, 341 (1979).

[2] M. Rahaman, J. Alam, Phys. Rev. C **97**, 054906 (2018).