Isospin mixing in the first $3^-$ state of $^{36}$Ar

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Introduction

$^{36}$Ar remains an exciting nucleus to understand the microscopic origins of shape coexistence, alpha-clustering [1, 2] and superdeformation [3]. Shell model calculations have been performed on $^{36}$Ar extensively in the past using several effective interactions [2, 3, 4, 5]. Usually, the positive parity states are explained satisfactorily in terms of 0, 2 or 4 $\hbar$o excitations, and the low-lying negative parity states are interpreted as 1 $\hbar$o excitations. However, in almost all the calculations, it was seen that the first $3^-$ state lies almost 1 MeV above the experimental state. This mismatch has been interpreted in terms of a large collectivity in the $3^+_1$ state [5]. In addition to this mismatch, this state also showed a strong signature of charge independence violation-decay via forbidden E1 transition. Isospin selection rules forbid the occurrence of $\Delta T=0$ E1 transitions in N=Z nuclei. However, different T states of the same spin parity may mix due to Coulomb and other isospin non-conserving interactions, and $\Delta T=1$ channels may open up, resulting in the observation of such forbidden transitions. This phenomenon is called isospin mixing. The $3^+_1$ state of $^{36}$Ar also decays via one such forbidden E1 transition. We have also found similar mismatch in energy for the $3^+_1$ state of $^{36}$Ar from our calculations done in the isosymmetric formalism. This over-prediction of the shell model state may, therefore, be due to isospin mixing. In the present work, we have investigated the origin of the $3^+_1$ state of $^{36}$Ar based on available experimental data and shell model calculation.

Experimental Observation

In $^{36}$Ar, the $3^+_1$ state at 4178 keV decays to the $2^+_1$ state at 1978 keV via an isospin-forbidden E1 transition, with a transition strength (B(E1)) of $2.3\times10^3(3)$ W.u. [6].

Theoretical Calculation

Large basis shell model calculations have been performed for the low lying $0^+, 2^+, 4^+, 6^+, 3^-, 5^-$ and $6^-$ states (both T=0 and 1) of $^{36}$Ar using the code OXBASH [7]. The valence space consists of sd and fp major shells above the $^{16}$O inert core for both protons and neutrons. The sd/pfmw interaction [8] has been used for the calculations, considering isospin as a good quantum number. The number of valance particles is 20. 0 $\hbar$o excitations within the sd shell have been used to reproduce the positive parity states. To reproduce the negative parity states, 1 $\hbar$o excitations within the full sd-fp model space have been considered. Mass normalization factor, defined for cross-shell terms, have been taken accordingly. The results for the calculations have been shown in Fig.1. It is seen from the figure that calculated excitation spectra for the positive parity states (Theo. pos) matches well with the experimental levels. For the negative parity states (Theo. neg), however, it is seen that apart from the $3^+_1$ state, all other states are in fair agreement with the experimental levels. The $3^+_1$ state is predicted ~1MeV above the experimental state.

Fig.1 Comparison between experimental and theoretical levels of $^{36}$Ar.

This mismatch may be removed by exciting more particles into the fp-shell. Therefore, we have done a 3 $\hbar$o calculation for the $3^-$ state. In this calculation, the $1d_{5/2}$ orbital has been filled up to 12 particles due to computational limitation. Following the work of ref. [2] on 4p-4h calculation, the SPE of each of the fp orbitals has

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been reduced by 3.375 MeV. In this calculation, 3\(^{-}\) state is predicted at 7.406 MeV (marked by an asterisk in Fig.1), \sim 3\text{MeV} above the experimental state. The next approach, in general, would be to do a mixed 1h0+3h0 calculation. Doing so may reproduce the energy of the state, but it will not justify the observation of E1 transition. So, we preferred to work with a different approach.

As the sd\textit{f}\textit{m}m interaction is formulated assuming strict charge independence, the mismatch of \sim 1 MeV between the theoretical and experimental \(3^-\)\(_T\) state might be a direct indication of the violation of charge independence. Experimental observation of the forbidden \(3^-\)\(_T\) to \(2^+\)\(_T\) E1 transition strongly supports this violation. Therefore, we opted to work within the framework of isospin mixing rather than a 1h0+3h0 mixing.

### Two-level Mixing Formalism

Due to coulomb and other isospin non-conserving interactions, the \(3^-\), T=0 state mixes with the \(3^-\), T=1 state. As a result, it is pushed down in energy and is observed experimentally \sim 1 MeV below. We have used the simple two-level formalism of ref [9] to explain this process. The wave function of the observed \(3^-\) state is written as a `mostly' T=0 state with a T=1 mixture,

\[
|3^-\text{, expt.} \rangle = \sqrt{1-b^2} |3^-\rangle_{T=0} - b |3^-\rangle_{T=1}
\]  

(1a)

The other member of this doublet is `mostly' T=1,

\[
|3^-\text{, expt.} \rangle = \sqrt{1-b^2} |3^-\rangle_{T=0} + b |3^-\rangle_{T=1}
\]  

(1b)

This state lies higher up in energy and is not observed in experiments. Since there is no mismatch in the \(2^+\) state, it can be written as a pure T=0 state,

\[
|2^+, \text{ expt.} \rangle = |2^+,T=0\rangle
\]  

(2)

The orthonormal basis sets in equations (1) and (2) are pure T=0 and T=1 states obtained from shell model diagonalization, and \(b\) is the isospin mixing amplitude. We next consider the E1 matrix element between the \(3^-\) and \(2^+\) states. In N=Z nuclei, the isovector part of the E1 operator does not contribute to \(\Delta T=0\) transition, leaving only an isoscalar part. The matrix element of the isoscalar part of the E1 operator also vanishes in the long-wavelength limit. Thus, the matrix element of the nuclear E1 operator vanishes when both parent and daughter states have the same isospin. The E1 transition matrix element between the \(3^-\) and \(2^+\) states is, therefore, equal to,

\[
(2^+\text{expt.}|E1|3^-,\text{expt.}) = -b(2^+,T=0)|E1|3^-,T=1.
\]  

(3)

The matrix element on the RHS of eq. (3) is calculated using the wave functions obtained from shell model calculation. While calculating this matrix element, effective charges, \(e_c=1.5\) and \(e_v=0.5\) were used. The matrix element on LHS is experimental.

### Results and Discussions

The value of isospin mixing amplitude, \(b^2\) calculated using eq. (3) is 17\%. This substantial value of isospin mixing amplitude means that the \(3^-\) state of \(^{36}\text{Ar}\) has a significant T=1 character. As a result, it is pushed down in energy by \sim 1 MeV.

From the isospin mixing amplitude, we have calculated the mixing matrix element as \((V)=2.12\ MeV\) [9]. We then diagonalized the matrix to calculate the mixed-isospin energy eigenvalues. The mixed-isospin energy for the \(3^-\) state is 4.084 MeV which is in excellent agreement with the experimental value of 4.178 MeV.

### Conclusion

The first \(3^-\) level in \(^{36}\text{Ar}\) has been interpreted as a mixed isospin state. The isospin mixing probability and the total mixing matrix element have been calculated. Calculated isospin-mixed energy eigenvalue is in excellent agreement with the experimental level energy.

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### References