Hadron-hadron scattering by velocity-dependent potential

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Introduction

Our understanding of the physical processes in the microscopic realm is largely best on quantum theory of scattering. The experimental information about the nucleon-nucleon and nucleon-nucleus systems is obtained from their scattering data. This is supplemented by the properties of the bound states of these systems. In the hadron-hadron scattering the recoil of the hadrons due to the exchanging particle can hardly be ignored and the interaction cannot be represented by one radial variable. Thus, such interactions are represented by non-local potentials $V(r,r')$ of two variables. The replacement of a phenomenological local potential by a non-local one in the sub-atomic region is no loss of generalization. The present text addresses itself to the construction of a phase equivalent velocity-dependent local potential corresponding to a local plus non-local interaction by a simple mathematical prescription.

Method of localization

The rank one Yamaguchi [1] potential with symmetric form factors is defined as

$$V(r,r') = \lambda\, g(r)\, g(r') = \lambda\, e^{-\beta r}\, e^{-\beta r'}, \tag{1}$$

where λ is the strength and β, the inverse range parameter. The Schrödinger equation for the Coulomb plus Yamaguchi [1] potential satisfies the integro-differential equation

$$\left[\frac{d^2}{dr^2} + k^2 - V_C(r)\right] u_{CY}(k,r) = \lambda\, g(r) \int_0^\infty dr'\, g(r')\, u_{CY}(k,r') \tag{2}$$

with

$$V_C(r) = \frac{2k\eta}{r}. \tag{3}$$

The quantity η is the Sommerfeld parameter. If $k\eta < 0$, it represents attraction and $k\eta > 0$ for repulsion.

The regular solution $u_{CY}(k,r)$ for the Coulomb plus Yamaguchi potential can be expanded in Taylor series [2] up to n=2 to get

$$\left[\frac{d^2}{dr^2} + k^2 - \frac{V_C(r)}{(1-G_2(r))}\right] u_{CY}(k,r) = V_1(k,r)\, u_{CY}(k,r) + V_2(r) \frac{d}{dr} u_{CY}(k,r) \tag{4}$$

where the quantities $V_1(k,r)$ and $V_2(r)$ are given by

$$V_1(k,r) = \frac{G_0(r) - k^2 G_2(r)}{1 - G_2(r)} \tag{5}$$

and

$$V_2(r) = \frac{G_1(r)}{1 - G_2(r)} \tag{6}$$

with

$$G_0(r) = \lambda\, e^{-\beta r} \int_0^\infty dr'\, e^{-\beta r'}, \tag{7}$$

$$G_1(r) = \lambda\, e^{-\beta r} \int_0^\infty dr'\, (r - r') e^{-\beta r'} \tag{8}$$

and

$$G_2(r) = \frac{\lambda\, e^{-\beta r}}{2} \int_0^\infty dr'\, (r' - r)^2\, e^{-\beta r'}. \tag{9}$$

The above quantities are given by

$$G_0(r) = \frac{\lambda}{\beta} e^{-\beta r}, \tag{10}$$

$$G_1(r) = \frac{\lambda}{\beta^2}(1-\beta r)\, e^{-\beta r} \tag{11}$$

and

$$G_2(r) = \frac{\lambda}{2\beta^3}\left(1+(\beta r-1)^{\,2}\right)e^{-\beta r}. \tag{12}$$

Thus, from Eqn. (4) one can identify the resultant energy-momentum dependent potential for the Coulomb plus Yamaguchi interaction as

$$V_{CY}(k,r) = \frac{V_C(r)}{(1-G_2(r))} + V_1(k,r) + V_2(r)\frac{d}{dr}. \tag{13}$$

The traditional phase function method of Calogero [3] is not applicable in this case. For such a velocity-dependent potentials it needs modification. This modification was first introduced by McKellar and May [4]. According to McKellar and May [4] the phase equations corresponding to equations (4) is written as

$$\delta'(k,r) = -k^{-1}\left[\left(\frac{V_C(r)}{(1-K_2(r))} + V_1(k,r)\right) \times \sin(kr+\delta(k,r)) + kV_2(r)\cos(kr+\delta(k,r))\right]. \sin(kr+\delta(k,r)) \tag{14}$$

The scattering phase shift $\delta(k)$ is achieved by solving the above equations from origin to asymptotic region with the initial condition $\delta(k,0)=0$. Finally, one gets the phase shift $\delta(k) = \lim_{r\to\infty} \delta(k,r)$.

Results and discussions

Table-1: Parameters for the non-local potentials

System	State	λ (fm^{-3})	β fm^{-1})
Ŭ-n	$(1/2)^+$	-10.8	1.229
Ŭ-p	$(1/2)^+$	-12.56	1.3318

With the parameters of Table-1 in conjunction with Eq. (14) we compute the scattering phase shifts for various systems under consideration and present them in Tables 2 & 3 for alpha-neutron and alpha-proton systems respectively. We have used $\hbar^2/2m = 25.92\, MeV\, fm^2$ for $\alpha-$nucleon systems. Looking at Tables 2 & 3 it is noticed that our phase shifts are in good agreement with those of Satchler et al. [5] over the entire energy range under consideration.

Table-2: Scattering phase shifts for (Ŭ-n) system.

E_{Lab} (MeV)	$ŭ^{1/2(+)}$ (degree)	Ref. [5]
0.3125	166.27	166.3
0.4375	163.88	164.2
0.8125	158.97	157.7
1.0	156.04	155.1
2.0	146.02	145.2
3.0	138.52	137.8
4.0	132.27	131.7
5.0	127.87	126.5
6.0	123.24	122.0
7.0	118.12	117.9
8.0	115.39	114.2
10.0	108.17	107.6
12.0	103.26	102.0

Table-3: Scattering phase shifts for (Ŭ-p) system.

E_{Lab} (MeV)	$ŭ^{1/2(+)}$ (degree)	Ref. [5]
1.56	159.03	161.5
2	156.19	157.5
3	151.74	149.1
4	144.19	142.3
5	132.09	136.7
6	125.54	131.1
7	122.93	127.2
8	119.12	123.3
10	112.53	116.2
14.31	100.42	104.2

Thus, we conclude that the present method deserves some attention to nuclear physicists.

References

[1] Y. Yamaguchi, Phys. Rev. **95**, 1628 (1954).
[2] B Talukdar, G C Sett and S R Bhattaru, J. Phys. G: Nucl. Phys. **11**, 591 (1985).
[3] F. Calogero, *Variable Phase Approach to Potential Scattering* (Academic, New York, 1967).
[4] B H J McKellar and R M May, Nucl. Phys. **65**, 289 (1965).
[5] G R Satchler et al. Nucl. Phys. A **112**, 1 (1968).