Test for validity of rotational energy formulae for SD bands

H. M. Mittal,* Monica Karday, and Anshul Dadwal
Department of Physics,
Dr. B. R. Ambedkar National Institute of Technology,
Jalandhar- 144011, INDIA

Introduction

The phenomenon of superdeformation was proposed to account for the fission isomers observed in some actinide nuclei. Strutinsky predicted the existence of superdeformation [1] and the first high-spin superdeformed (SD) band was discovered in ^{152}Dy by Twin *et al.* [2]. Since then many SD bands have been observed and intensively studied in $A \sim$ 190, 150, 130 and 80 mass regions. The SD bands near $A \sim 190$ region show smooth and large increase of the dynamic MoI $\Im^{(2)}$ with rotational frequency $\hbar\omega$ whereas the SD bands near $A \sim 190$ show pronounced variations in $\Im^{(2)}$. A systematic analysis of identical SD bands and role of pairing correlations was analyzed [3, 4]. The band-head spins have been established experimentally for the SD bands $^{194}Hg(1,3)$, $^{193}Tl(1,2)$, ^{192}Pb, $^{194}Pb(1)$, $^{196}Pb(1,2,3)$ in $A \sim 190$ region and for $^{152}Dy(1,6)$ in the $A \sim 150$ region. These bands with firmly established spins can prove out to be excellent testing grounds for the theories. In the present analysis, we have employed several rotational energy formulae to deduce the band-head spins of the above mentioned SD bands and to analyse the validity of rotational energy formulae for the SD bands in $A \sim 190$ and $A \sim 150$ mass regions.

The Rotational Energy Formulae

We have used various rotational energy formulae like variable moment of inertia (VMI) model [5], ab formula [6], Harris ω^2 expansion [7], Exponential model [8, 9] and Nuclear softness formula [10] to study the SD bands in $A \sim 190$ and $A \sim 150$ regions in order to test the validity of various rotational energy formulae in different mass regions of SD bands.

1) VMI model:

$$E_I = E_0 + \left[\frac{I(I+1)}{2\Im_0}\right]\left[1 + \frac{I(I+1)}{4C(\Im_0)^3}\right] \quad (1)$$

2) ab Formula:

$$E = a\left[\sqrt{1 + bI(I+1)} - 1\right] \quad (2)$$

3) Harris ω^2 expansion:

$$\Im^{(2)} = A + B\omega^2 + C\omega^4 \quad (3)$$

4) Exponential model with pairing attenuation:

$$E(I) = \frac{\hbar^2}{2\Im_{(Expo)}} I(I+1) e^{\left[\triangle_0\left(1-\frac{I}{I_c}\right)^{1/2}\right]} \quad (4)$$

Modified exponential model:

$$E(I) = \frac{\hbar^2}{2\Im_{(Expo)}} I(I+1) e^{\left[\triangle_0\left(1-\frac{I}{I_c}\right)^{1/\nu}\right]} \quad (5)$$

5) Nuclear softness formula:

$$E = \frac{\hbar^2}{2\Im_0} \times \frac{I(I+1)}{(1 + \sigma_1 I + \sigma_2 I^2)} \quad (6)$$

Keeping only σ_1 in Eq. 6, we get NS(2) model. By incorporating σ_2 parameter in Eq. 6, we obtain NS(3) model.

*Electronic address: `mittalhm@nitj.ac.in`

TABLE I: The calculated fitting parameters for the nine SD bands in $A \sim 190$ and two SD bands in $A \sim 150$ using the exponential and modified exponential model.

		Exponential model			Modified Exponential model				Ref [11]
SD Band	E_γ^{exp} (keV)	I_0 ($\hbar$)	$\Im_0$ ($\hbar^2 MeV^{-1}$)	$\triangle_0$	I_0 ($\hbar$)	$\Im_0$ ($\hbar^2 MeV^{-1}$)	$\triangle_0$	ν	I_0 ($\hbar$)
$^{194}Hg(1)$	211.7	8	87.52	0.41	8	86.86	0.96	4.99	8
$^{194}Hg(3)$	222.0	9	91.86	0.32	9	92.06	0.30	1.98	9
$^{193}Tl(1)$	206.6	8.5	93.42	0.30	8.5	93.58	0.33	2.31	8.5
$^{193}Tl(2)$	227.3	9.5	94.06	0.25	8.5	82.70	0.22	0.53	9.5
$^{192}Pb(1)$	214.8	8	83.60	0.52	7	70.96	0.33	0.43	8
$^{194}Pb(1)$	124.9	4	86.15	0.42	4	85.20	0.28	1.10	4
$^{196}Pb(1)$	171.4	6	84.32	0.41	6	84.84	0.73	4.04	6
$^{196}Pb(2)$	204.5	8	89.61	0.34	7	78.24	0.26	0.49	8
$^{196}Pb(3)$	226.7	9	89.73	0.34	9	89.98	0.45	2.83	9
$^{152}Dy(1)$	602.4	24	84.62	0.02	24	84.68	0.18	0.22	24
$^{152}Dy(6)$	761.5	32	87.75	0.01	31	85.36	0.15	0.30	31

Results and Discussion

The observed transition energies of the SD bands $^{194}Hg(1,3)$, $^{193}Tl(1,2)$, ^{192}Pb, $^{194}Pb(1)$, $^{196}Pb(1,2,3)$, and $^{152}Dy(1,6)$ [11, 12] have been fitted to the rotational energy formulae and the corresponding fitting parameters are calculated. For reference, the fitting parameters along with the calculated spins for the exponential and modified exponential model have been displayed in Table I. We observe that the pairing gap parameter $\triangle_0$ is larger for $A \sim 190$ region as compared to $A \sim 150$ region which is justified due to presence of large shell gaps in $A \sim 150$ region. However, the exponential model predicts a negligible $\triangle_0$ for SD bands in $A \sim 150$ region whereas the modified exponential model gives a small but finite value of $\triangle_0$ in $A \sim 150$. Further, the NS(2) formula is found to give reasonable results in $A \sim 190$ region, while after adding an additional parameter the NS(3) formula gives better agreement with experiment for $A \sim 150$ region.

Conclusion

In the present analysis, it has been observed that the modified exponential model and NS(3) gives better results for the SD bands in $A \sim 150$ as compared to other models, while the exponential model and NS(2) works better in $A \sim 190$ region. The present study suggests that some perturbations to the rotational energy expressions are required when we move from SD bands in $A \sim 190$ mass region to the $A \sim 150$ mass region.

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