Garvey-Kelson relations revisited for Masses of Nuclei far from the valley of stability

B.Pfeiffer[1], D.S. Shreesha Rao[2,3], K. Vijay Sai[2], K. Venkataramaniah[2*], C. Scheidenberger [1]

[1]GSI Helmholtzzentrum f¨ur Schwerionenforschung, Darmstadt, Germany

[2]Laboratories for Nuclear Research, Department of Physics, Sri Sathya Sai Institute of Higher Learning, Prasanthinilayam 515134, INDIA

[3]DTU Fotonic, Department of Photonics Engineering, Technical University of Denmark, Oested Plads,2800 Kongens Lyngby, Denmark

* email: vrkamisetti@gmail.com

Introduction

A great deal of work has been done to obtain formulas that will give the masses of all particle-stable nuclides in the periodic table. Such formulas, based usually on the masses of known nuclides, may be used to predict the masses of undiscovered isotopes. These global mass formulas, however, will in general be too approximate to make predictions to a sufficient degree of accuracy.

Therefore, simple relations between neighbouring nuclidic masses were developed which will be valid, independently of the actual variation of mass with atomic number and charge. Such relations will not replace mass formulas, but may be very useful because of their simplicity and greater accuracy in making ground-state mass predictions.

In the case of lighter nuclei, the Isobaric Multiplet Mass Equation (IMME) is a powerful interpolation/extrapolation instrument 3. Recent high-precision mass measurements have led to a renewed interest in this technique (see, i.e. [1,2]). Already back in 1954, Way and Wood [3] derived graphical presentations of nuclear decay data on the basis of the semi-empirical Bethe-Weizs¨acker mass formula [4,5]. In the absence of nuclear structure effects as magic numbers, the lines connecting nuclei with constant N or Z ought be “straight” lines. Under the assumption that the atomic masses are smoothly varying functions of N and Z (as the semi-empirical mass formula) the lines can be extrapolated to unmeasured nuclei.

Methodology:

In 1966 Garvey and Kelson (G-K) presented such a relation based on an independent particle model of the nucleus [6]:

$$M(N+2,Z-2)-M(N,Z)+M(N,Z-1)-M(N+1,Z-2)+M(N+1,Z)-M(N+2,Z-1) = 0 \quad (1)$$

This equation relates six nuclides. With five known masses, a sixth unknown one can be predicted .

The applicability of the method has been tested with cases in which either all six mass excess values of Equ. (1) are known experimentally or five values have been determined experimentally and the sixth value can be estimated and compared with the recent experimental values or extrapolated masses from AME12 [8] or AMC12 [9]. Table 1 lists examples taken from the values calculated. It is interesting to note that G-K relations could predicted the unknown masses with the same accuracy as those of recent experimental measurements as well the our extrapolated values using weighted mean method and those of AME12 where a different extrapolation method was used suggesting that G-K relations are still relevant if the masses of the five neighboring nuclei are available with high precision.

Results and discussion

Although the size of the sample is statistically insufficient one may conclude that the assumptions underlying Equ. (1) are fulfilled with uncertainties in the range of a few hundred keV for isotopes close to the measured ones. It should be mentioned that Garvey and Kelson express a warning against neglecting nuclear structure effects not represented by the independent-particle but that the inclusion of such structural complexity would require a far more elaborate and sophisticated approach. "It is independent-particle but that the inclusion of such structural complexity would require a far more elaborate and sophisticated approach. They close their 1966 Letter with the sentence: "It is our feeling, nevertheless, that it is a useful approximation and should be exploited in ground-state energy predictions. In recent times, there are new investigations into features of these relations, as p.e. [10,11]. . It has to be remarked that some experimental mass excess values are only known with an uncertainty greater than 100 keV, going up to 450 keV in the case of ^{93}Br. A meaningful test has to await further decreases in the measuring uncertainties for very unstable nuclei.

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Table1: Comparison of Mass Excess values calculated from Gravey-Kelson relations and the extrapolated values from AME12 [8] and AMC12[9]

Isoto -pe	Formula	G-K Extrap-olation (keV)	AMC12 Data and Extrapol-ation (keV)	GK-AMC1 2 (keV)	AME12 Data and Extrapolation (keV)	GK-AME12 (keV)
^{83}Ga	^{83}As +^{82}Ga -^{82}Ge +^{84}Ge -^{84}As	-49479	-49257(26)*	222	-49257(26)$^{+}$	222
^{87}As	^{87}Br +^{86}As -^{86}Se +^{88}Se -^{88}Br	-55516	-55618(30)*	-102	-55618(30)$^{+}$	-102
^{103}Y	^{103}Nb +^{102}Y -^{102}Zr +^{104}Zr -^{104}Nb	-58509	-58458(18)*	51	-58458(11)$^{+}$	51
^{84}Ga	^{84}As +^{83}Ga -^{83}Ge +^{85}Ge -^{85}As	-44069	-44180(360)	-111	-44280(400)	-211
^{86}Ge	^{86}Se +^{85}Ge -^{85}As +^{87}As -^{87}Se	-49626	-49760(300)	-134	-49760(300)	-134
^{88}As	^{88}Br +^{87}As -^{87}Se +^{89}Se -^{89}Br	-50626	-50432(50)	194	-50720(200)	-288
^{91}Se	^{91}Kr +^{90}Se -^{90}Br +^{92}Br -^{92}Kr	-50238	-50300(110)	-62	-50340(500)	-102
^{94}Br	^{94}Rb +^{93}Br -^{93}Kr +^{95}Kr -^{95}Rb	-47662	-47580(150)	82	-47600(400)	62
^{98}Kr	^{98}Sr +^{97}Kr -^{97}Rb +^{99}Rb -^{99}Sr	-44025	-43860(310)	165	-44310(300)	-285
^{100}Rb	^{100}Y +^{99}Rb -^{99}Sr +^{101}Sr -^{101}Y	-46524	-46650(150)	-126	-46550(200)	-26
^{104}Y	^{104}Nb +^{103}Y -^{103}Zr +^{105}Zr -^{105}Nb	-54021	-53968(150)	53	-54060(400)	-39

* Experimental Mass excess values from AMC12 $^{+}$ Experimental Mass excess values from AMC12. Experimental mass excess values from AMC12 have been used in the G-K formula

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