Hyper-radius formalism for the ternary mass distribution

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Introduction

The mass distribution studies in binary and ternary fission phenomena helps to understand the underlying physics in fragment formation. The binary fission studies of radioactive nuclei has been well studied using preformed cluster model (PCM) [1] of Gupta and collaborators. As an extension to PCM, three cluster model (TCM) [2] has been proposed to explain the particle accompanied fission. TCM relies on the potential energy surface calculation based on surface-surface separation between three fragments. Ternary yield distribution was qualitatively illustrated based on Quantum mechanical fragmentation theory and by WKB approximation using the calculated potential energy. As an alternative to surface separation, we made an attempt to study ternary mass distribution using hyperradius formalism. The mechanism for the equatorial three body decay in the spontaneous ternary fission of ^{252}Cf with ^{4}He as third particle is presented here.

Model

The hyperspherical adiabatic method is fairly accurate for three body decay with only the dominating potential [3]. The generalized effective radial potentials are expressed as function of the hyperradius, ρ defined by,

$$\rho^2 \equiv \frac{1}{mM} = \sum_{i<k} m_i m_k r_{ik}^2 = \left(X_j^2 + Y_j^2\right), \tag{1}$$

with $r_{ik}^2 = (r_i - r_k)^2$ where (i, j, k) is a permutation of $(1, 2, 3)$, r_i is the coordinate of particle i, $M = \sum m_i$ and m is an arbitrary normalization mass, X_j and Y_j are respectively proportional to the distance between two particles and the distance between their centre of mass and the third particle.

The effective potential responsible for three body decay is defined as,

$$V(\rho) = \sum_{i=1}^{3} M_i + \sum_{i=1}^{3}\sum_{j>i}^{3}(V_{Cij} + V_{Nij}), \tag{2}$$

where the first term is the sum total of mass excess of all the three fragments taken from mass table [4]. The Coulomb term is defined as, $V_{Cij} = \frac{Z_i Z_j e^2}{r_{ij}(\rho)}$ with $r_{ij}(\rho)$ is the distance between the centres of the fragments A_i and A_j evaluated from Eq. (1). The proximity potential [5] is taken as the nuclear potential and is defined as,

$$V_{Nij} = 4\pi \bar{R}_{ij} \gamma b \phi(\varepsilon), \tag{3}$$

with $\bar{R}_{ij} = \frac{R_i R_j}{R_i + R_j}$ and the radius expression R_i is calculated using, $R_i = 1.28 A_i^{1/3} - 0.76 + 0.8 A_i^{-1/3}$. The universal function $\phi(\varepsilon)$ independent of the geometry of the system defined as,

$$\phi(\varepsilon) = \begin{cases} -\frac{1}{2}(\varepsilon - k)^2 - 0.085(\varepsilon - k)^3, \varepsilon < 1.251 \\ -3.437 \exp(-\varepsilon/0.75), \varepsilon \geq 1.251 \end{cases} \tag{4}$$

with k=2.54 and γ is the nuclear surface energy coefficient given by,

$$\gamma = 0.952 \left[1 - 1.783 \left(\frac{N - Z}{A}\right)^2\right] MeV fm^{-2}, \tag{5}$$

where $\varepsilon = \frac{r_{ij}(\rho) - R_i - R_j}{b}$ is the separation distance between the two surfaces in units of b.

For an effective hyperradial potential, WKB tunnelling transmission P can be,

$$P = \exp\left(-\frac{2}{\hbar}\int_{\rho_0}^{\rho_t} d\rho \sqrt{2\mu(V(\rho) - E)}\right), \tag{6}$$

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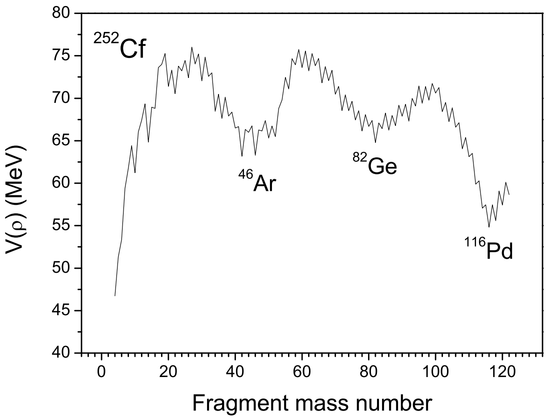

FIG. 1: The total hyperradius potential for the charge minimized fragment combinations for the ternary breakup of ^{252}Cf with fixed third fragment ^{4}He.

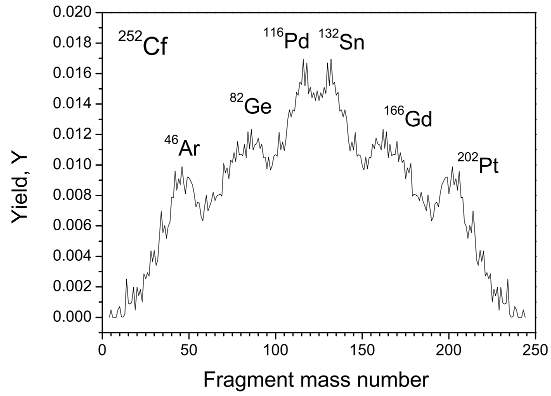

FIG. 2: The transmission probability for the direct three body equatorial decay of ^{252}Cf with ^{4}He as fixed third fragment.

where E is the total energy of the system after separation, ρ_0 and ρ_t are the classical turning points where $V(\rho_0) = V(\rho_t) = E$ and μ is the reduced mass of the system given by

$$\mu = \frac{(A_1 A_2 A_3)}{A_1 A_2 + A_2 A_3 + A_1 A_3} m_n, \quad (7)$$

with m_n being the nucleon mass.

The relative yields for all the charge-minimized fragmentation channels are calculated as the ratio between the penetration probability of a given fragment over the sum of penetration probabilities of all possible fragmentation as,

$$Y(A_i, Z_i) = \frac{P(A_i, Z_i)}{\sum P(A_i, Z_i)}. \quad (8)$$

The yield values are normalized to two.

Results and Discussion

The ternary combinations of ^{252}Cf with fixed third fragment ^{4}He were generated by potential energy minimization with respect to charge. The potential energy surface for the charge minimized combinations is shown in Fig. 1. The relative yield of the minimized combinations evaluated using Eq. (8) is presented in Fig. 2. The combinations with minimum in potential energy posses maximum yield values which are labelled. The preferable ternary breakup is found to associate with proton or neutron closed shell in any one of the three fragments. The nuclei which were found to exhibit maximum yield which includes ^{132}Sn, ^{82}Ge and ^{46}Ar where these nuclei possess closed shell proton or neutron number or both. The model can be improved with consideration of deformation and orientation degrees of freedom with appropriate nucleus-nucleus interaction; it is planned to present the results with deformation degree of freedom as well. The method of hyperradius potential can be used as a good tool to study three body decay of any given system.

References

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