Pion-photon mixing in magnetized vacuum

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Introduction

The neutral pions (π_0) having $J^p = 0^-$, couples anomalously to photon (γ) by dim-5 operator of the following form ,

$$L_{eff} = \frac{N_c e^2}{96\pi^2 f_\pi}\pi^0 \epsilon^{\mu\nu\lambda\rho} F_{\mu\nu} F^{\lambda\rho} \quad (1)$$

where, we can define $g_{\pi\gamma\gamma} = \frac{N_c e^2}{96\pi^2 f_\pi}$. N_c here is equal to three, and rest of thr terms have their usual meaning. Their interaction with photon (γ) through dimension-five operators of [1] has been discussed in [2–4]. π^0 interaction with photon (γ) in the vicinity of compact astrophysical sources of can be an useful tool to study the magnetosphere physics of compact objects .

In an external background magnetic field$\bar{F}_{\mu\nu}$, (when $\bar{F}_{12} \neq 0$,), out of the two polarization states of photon, $\bar{F}f$ (**CP** symmetric) and $\tilde{\bar{F}}f$ (**CP** violating). only $\tilde{\bar{F}}f$ couples to π and ($\bar{F}f$) remains free. Therefore like neutrino flavour oscillation , π^0 γ ($\tilde{\bar{F}}f$) oscillation can be realized in an external field. In magnetized vacuum, photon – pion conversion rate, as a function of travelled distance z is given by [1];

$$P(z) = \frac{4\omega^2 B^2}{m_\pi g_{\pi\gamma\gamma}^{-2} + 4\omega^2 B^2} sin^2\left(\frac{\sqrt{m_\pi g_{\pi\gamma\gamma}^{-2} + 4\omega^2 B^2}}{4\omega g_{\pi\gamma\gamma}^{-1}} z\right) \quad (2)$$

hence, even if there is no pion to begin with, due to photon pion conversion probablity P (z) [2] there would be non-zero no density of pions in the magnetosphere of compact objects due to curvature radiation. Presence of these pions make the magnetosphere optically active, that can be tested with high energy photons.

Equations of motion

In presence of a magnetic field $\tilde{F}^{\mu\nu}$, the Lagrangian for $\pi - \gamma$ turns out to be,

$$\begin{aligned} L &= \frac{1}{2}[(\partial_\mu^2 \pi) - m_\pi^2] - \frac{1}{4} f_{\mu\nu} f^{\mu\nu} \\ &- \frac{1}{2} g_{\pi\gamma\gamma} \pi \tilde{\bar{F}}^{\mu\nu} f_{\mu\nu} - \frac{1}{4}\tilde{f}_{\mu\nu} f^{\mu\nu} \end{aligned} \quad (3)$$

And the equation of motion for the two degree of freedom of photon:

$$\begin{aligned} (\partial_\mu \partial^\mu - m_\pi^2)\pi - \frac{g_{\pi\gamma\gamma}}{2}\tilde{\bar{F}}_{\mu\nu} f^{\mu\nu} &= 0 \\ \partial_\mu f^{\mu\nu} - g_{\pi\gamma\gamma}\tilde{\bar{F}}^{\mu\nu}\partial_\mu \pi &= 0 \end{aligned} \quad (4)$$

In terms of guage invariant operators, the same can be expressed in the form as,

$$\begin{aligned} \partial^2\left(\frac{\tilde{\bar{F}}^{\lambda\beta} f_{\lambda\beta}}{2}\right) - g_{\pi\gamma\gamma}(\partial_\lambda \partial^\mu \pi)(\tilde{\bar{F}}^{\lambda\nu}\tilde{\bar{F}}_{\mu\nu}) &= 0 \\ (\partial^2 - m_\pi^2)\pi - g_{\pi\gamma\gamma}\left(\frac{\tilde{\bar{F}}^{\mu\nu} f_{\mu\nu}}{2}\right) &= 0 \\ \partial^2\left(\frac{\bar{F}^{\mu\nu} f_{\mu\nu}}{2}\right) &= 0 \end{aligned} \quad (5)$$

Transforming the above three equations of motion for $\pi-\gamma$ coupling in momentum space one can write the same in matrix form as,

$$\begin{pmatrix} k^2 & 0 & 0 \\ 0 & k^2 & -g_{\pi\gamma\gamma}\omega B_T \\ 0 & -g_{\pi\gamma\gamma}\omega B_T & k^2 - m^2 \end{pmatrix} \begin{pmatrix} \psi \\ \tilde{\psi} \\ \Pi \end{pmatrix} = 0. (6)$$

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%beginwidetext From the equations of nmotion it can be verified that ψ, $(\Pi\sin\theta + \tilde{\psi}\cos\theta)$ and $(\ \Pi\cos\theta - \tilde{\psi}\sin\theta)$ (when $theta = \frac{1}{2}tan^{-1}\left(\frac{2g_{\pi\gamma\gamma}B_T\omega}{m_\pi^2}\right)$) satisfies the following dispersion relations given as follows:

$$\begin{aligned} \omega &= k, \\ \omega_+ &= \pm\sqrt{k^2 - \frac{m^2}{2} - \sqrt{\xi}}, \\ \omega_- &= \pm\sqrt{k^2 - \frac{m^2}{2} + \sqrt{\xi}}, \end{aligned} \quad (7)$$

The solutions for the dynamical variables turn out to be:

$$\begin{aligned} \psi(t,x) &= A_0 e^{i(\omega t - k.x)}, \\ \tilde{\psi}(t,x) &= A_1\cos\theta e^{i(\omega_+ t - k.x)} \\ &- A_2\sin\theta e^{i(\omega_- t - k.x)}, \\ \Pi(t,x) &= A_1\sin\theta e^{i(\omega_+ t - k.x)} \\ &+ A_2 cos\theta e^{i(\omega_- t - k.x)}. \end{aligned} \quad (8)$$

One can impose the boundary conditions $\Pi(0,0) = 0$ and $\psi(0,0) = 1$ on the dynamical degrees of freedom, to identify the constants A_0, A_1 and A_2 .

Stokes parameters

Using the solutions (8) for field in magnetized vacuum, the expressions for the stokes parameters $(\mathbf{I}, \mathbf{Q}, \mathbf{U}, \mathbf{V})$ obtained as follows:

$$\begin{aligned} I(\omega,z) &= 1 + \sin^4(\theta) + \cos^4(\theta) \\ &+ 0.5\sin^2(\theta)\cos\left[(\omega_+ + \omega_-)\,z\right] \quad (9) \\ Q(\omega,z) &= -1 + \sin^4(\theta) - \cos^4(\theta) \\ &+ 0.5\sin^2(\theta)\cos\left[(\omega_+ - \omega_-)\,z\right] \quad (10) \\ U(\omega,z) &= 2\cos^2(\theta)\cos\left[(\omega_+ - \omega)\,z\right], \\ &+ 2\sin^2(\theta)\cos\left[(\omega_- - \omega)\,z\right] \quad (11) \\ V(\omega,z) &= -2\cos^2(\theta)\cos\left[(\omega_+ - \omega)\,z\right], \\ &- 2\sin^2(\theta)\cos\left[(\omega_- - \omega)\,z\right] \quad (12) \end{aligned}$$

In the slot of gap model of compact stars, electrons (or charge particles) can be accelerated to energies amounting to GeV or more. These electrons can may emit photons having energy between 10 to 100 GeV, , some of them may convert into π^0s in the slot gap of the compact stars and contribute to optical activity. The polarization or the ellipticity angle for the same, for propagation length z can be estimated using eqns. (9) to (12). Considering the propagation length to be of the order of decay length of the pions, given by $z = \frac{4}{\pi g^2_{\pi\gamma\gamma}}$, we have estimated the polarization angle of transverse photons $(\tilde{\tilde{F}}^{\mu\nu} f_{\mu\nu})$ in the limit $\omega > m_\pi >> B_T >$. The estimate of the polarisation angle to order $\frac{1}{\omega}$ turns out to be :

$$\Psi = \frac{1}{2}\tan^{-1}\left[\frac{m_\pi^5}{\omega B_T^2}\right] \quad (13)$$

This tells us that , any departure of the polarisation angle for high energy photon $(\omega >> m_\pi)$, from this value , if detected from astrophysical sources, then one should look for an alternative explanation for for the origin of this polarisation. More about it will be discussed else-where.

References

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