Tidal Deformability in Compact Star Merger Events

Sujan Kumar Roy[1*§], Joydev Lahiri[2*] and Somnath Mukhopadhyay[3†]
[*] *Variable Energy Cyclotron Centre, 1/AF Bidhan Nagar, Kolkata 700 064, India*
[§] *Homi Bhabha National Institute, Training School Complex, Anushakti Nagar, Mumbai 400085 and*
[†] *BITS Pilani, Hyderabad Campus, Jawahar Nagar,*
Kapra Mandal, Medchal District 500078 Telangana, India[*]

Introduction

Application of density dependent M3Y interaction based on β-equilibrated infinite nuclear matter has been succesful in producing the mass-radius relationship for Neutron Stars. After commissioning of advanced LIGO interferometer several gravitational wave detection incidents have been reported. These incidents comprise of mergers of black hole and other compact star binaries in which the system emits copious amount of gravitational wave due to the "inspiral". It is worth mentioning the fact that, more compact is the star shorter will be the signal in the detectors. Compact Binary Inspiral gravitational waves vary in duration depending on the masses of the objects involved. Colliding black holes produce characteristically short gravitational waves on the order of fractions of a second, whereas neutron stars (being less compact than black holes) generate signals of several tens of seconds long and merging white dwarfs, expectedly, would produce even longer signals. In all the cases, the signal frequency increases rapidly as the objects spiral into each other, orbiting ever-faster. One such reported event was GW170817[1], a merger of two compact neutron stars in a binary system and the signal lasted for ∼ 30 Sec. There are other effects as well for which a system can emit wave, such as the distortion in the mass distribution.

As Gravitational field has a non-vanishing second derivative any test body subjected to a gravitational field experiences an additional force, characteristically which stretches along the line of joining of the field producing body and the test body and compresses in the perpendicular direction, is called Tidal Field. This field in leading order, varies inversely with the cubic power of the distance between the objects and directly with the dimension of the test body. When an inspiral occurs, components of a binary system get closer due to the emission of the gravitational wave, thereby results in an increase in the tidal field. This effectively deforms each of the stars due to the effect of the other and thus, each star acquires a quadrupole moment. This distortion in the mass distribution of the binary configuration contributes to the emission, by changing the phase evolution of the emitted wave. Induced quadrupole moment depends directly on the deformability parameter and the tidal field. As white dwarfs are less compact than neutron stars they are more vulnerable to applied external field. Due to the larger radii white dwrafs are subjected to larger tidal fields compared to neutron stars. Hence, a same binary companion effectively induces larger quadrupole moment in white dwarfs. Consequently, it can be expected that, coalescing white dwarfs will leave different imprints on the detectors both in sense of phase evolution and time duration of the signal.

This signature of the acquired quadrupole moment increases with time, as can be expected, and becomes considerably large more or less ∼ 15-17 mins prior to the merger. The detector signals can be analysed to evaluate the phase evolution and to reconstruct the wavefront. As how much a star will be deformed, depends on its structure i.e. on the nature of the equation of state(EoS), this analysis effectively constrains the properties of the EoS for compact stars. GW170817 has been reported with 150−800 Hz range of frequency and strain tensor amplitude $h_0 \sim 10^{-20}$ [1] with the advanced detectors in LIGO sites. Several recent studies have been carried out[2, 3] regarding the merger of neutron stars. In Ref.[2] 28 EoSs with different Skyrme interaction parameters have been used and Ref.[3] used differnet relativistic mean field EoSs. The results obtained from GW170817 event has been compared in both the references to finally constrain the EoSs, though the moment of inertia-tidal love number-quadrupole moment realtionship has been reported to be EoS independent [4]. But recently the non validity of this relationship has been reported in case of hot white dwarf[5]. Our aim is to report the tidal deformabilities of the neutron stars using the DDM3Y EoS and compare the calculated results with the observed ones to finally infer about the EoS compatibility. On the other hand, the validity of the moment of inertia-tidal love number-quadrupole moment relationship will also be checked and reported for magnetized white dwarfs.

Calculations and Results

Density dependent M3Y interaction based on β-equilibrated infinite nuclear matter has been used as neutron star EoS as described in Ref.[6] and for magnetized white dwarf EoS prescription used in Ref.[7] has been followed. Using Regge-Wheeler gauge the leading order perturbation H(r) caused by the tidal field can be included in the metric as follows:

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[*] E-mail 1: `sujan.kr@vecc.gov.in`; E-mail 2: `joy@vecc.gov.in`; E-mail 3: `somnathm@hyderabad.bits-pilani.ac.in`

TABLE I: Variation of tidal deformability of neutron star.

ρ (r=0) (fm^{-3})	Radius (Kms)	Mass ($M_\odot$)	k_2
1.31	9.75	1.92	1.88×10^{-2}
1.30	9.77	1.92	1.90×10^{-2}
1.20	9.99	1.91	2.18×10^{-2}
1.10	10.23	1.90	2.51×10^{-2}
1.00	10.48	1.87	3.00×10^{-2}
0.90	10.74	1.81	3.66×10^{-2}
0.80	11.01	1.73	4.49×10^{-2}
0.70	11.27	1.60	5.65×10^{-2}
0.60	11.51	1.43	7.19×10^{-2}

TABLE II: Variation of tidal deformability of magnetized Iron white dwarf. The central density has been taken to be so as to maintain β-stability at r=0.

B_d (r=0) (B_c)	Radius (Kms)	Mass ($M_\odot$)	k_2
1	2194.68	1.14	4.60×10^{-2}
2	2207.06	1.22	4.88×10^{-2}
3	2232.22	1.38	5.23×10^{-2}
4	2110.46	1.59	5.68×10^{-2}
5	2333.45	1.90	6.28×10^{-2}
6	2412.16	2.30	6.90×10^{-2}
7	2508.22	2.80	7.50×10^{-2}
8	2618.53	3.41	8.10×10^{-2}

$$\begin{aligned} ds^2 = & -e^{2\Phi(r)}[1 + H(r)Y_{20}(\theta,\phi)]dt^2 \\ & +e^{2\Lambda(r)}[1 - H(r)Y_{20}(\theta,\phi)]dr^2 \\ & +r^2[1 - K(r)Y_{20}(\theta,\phi)](d\theta^2 + \sin^2\theta d\phi^2) \end{aligned} \tag{1}$$

where, $K(r)$ is defined as $K^{'}(r) = H^{'}(r) + 2H(r)\Phi^{'}(r)$. Using the above metric tensor one would find the hydrostatic equations of equilibrium to be:

$$e^{2\Lambda} = \left(1 - \frac{2m(r)}{r}\right)^{-1} \tag{2}$$

$$\frac{d\Phi}{dr} = -\frac{1}{\epsilon + p}\frac{dp}{dr} \tag{3}$$

$$\frac{dp}{dr} = -(\epsilon + p)\frac{m(r) + 4\pi r^3 p}{r(r - 2m())} \tag{4}$$

$$\frac{dm(r)}{dr} = 4\pi r^2 \epsilon \tag{5}$$

$$\begin{aligned} & \left(-6\frac{e^{2\Lambda}}{r^2} - 2\left(\frac{d\Phi}{dr}\right)^2 + 2\frac{d^2\Phi}{dr^2} + \frac{3}{r}\frac{d\Lambda}{dr} + \frac{7}{r}\frac{d\Phi}{dr}\right)H(r) \\ & + \left(-2\frac{d\Phi}{dr}\frac{d\Lambda}{dr} + \frac{d\epsilon/dp}{r}\left(\frac{d\Phi}{dr} + \frac{d\Lambda}{dr}\right)\right)H(r) \\ & + \left(\frac{2}{r} + \frac{d\Phi}{dr} - \frac{d\Lambda}{dr}\right)\frac{dH}{dr} + \frac{d^2H}{dr^2} = 0 \end{aligned} \tag{6}$$

where $\epsilon(r)$, $p(r)$, $m(r)$ are total energy density, pressure density and mass of the sphere of radius r, respectively. Eqn.(2-6) are simultaneously solved with proper boundary conditions to find out the tidal deformability parameter. The tidal love number k_2 for leading order perturbation is defined as follows:

$$\begin{aligned} k_2 = & \frac{8C^5}{5}[2 + 2C(y-1) - y] \\ & \times [F_1(y) + F_2(y) + F_3(y)]^{-1} \end{aligned} \tag{7}$$

$$\begin{aligned} & F_1(y) = 2C[6 - 3y + 3C(5y - 8)] \\ & F_1(y) = 4C^3[13 - 11y + C(3y - 2) + 2C^2(1 + y)] \\ & F_1(y) = 3(1 - 2C)^2[2 - y + 2C(y - 1)]ln(1 - 2C) \end{aligned} \tag{8}$$

The so far obtained results have been presented in Table-I & II.

References

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