

# Gravitational waves from non-radial perturbations in neutron stars

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## Introduction

When a neutron star accretes mass from its companion, angular momentum gets transferred and consequently its angular velocity increases and exceeds its critical value  $\Omega_c$ . At this point the neutron star starts emitting gravitational wave (GW) caused from the perturbation of r-mode. Due to Chandrasekhar-Friedman-Schutz(CFS) mechanism [1, 2], GW emission pumps up the r-mode perturbation causing increase in amplitude  $\alpha_r$  till it saturates. The saturation value can be estimated either from ‘spin equilibrium’ or from ‘thermal equilibrium’ described earlier. The emitted GW carries with it the angular momentum and energy and the star spins down to the region of stability. In the present calculations we complement the investigations of [3] by estimating the intensity of the GW emitted by the NSs. This is expressed in terms of the amplitude of the strain tensor,  $h_0$ . The amplitude of the strain tensor,  $h_0$  is related to the r-mode amplitude  $\alpha_r$  by [4, 5]

$$h_0 = \sqrt{\frac{8\pi}{5}} \frac{G}{c^5} \frac{1}{r} \alpha_r \omega^3 M R^3 \tilde{J}, \quad (1)$$

where  $\omega$  is the r-mode angular frequency, which is related to the angular velocity of the star  $\Omega$  by the relation  $\omega = -\frac{(l-1)(l+2)}{l+1}\Omega$ . It is worth mentioning the fact that the signature of continuous GW emitted due to r-mode perturbation is different from those emitted due to the ellipticity of the star as discussed broadly in Ref.[5]. In the former case the radiation is dominated by the mass quadrupole moment whereas in the latter case it is dominated by the mass current quadrupolar moment i.e the gravitomagnetic effect and results into a phase difference between the components of these two type of events.

Where  $\alpha_r$  has been estimated from ‘thermal equilibrium’ the red-shifted effective surface temperature  $T_{eff}^\infty = 100$  eV has been used which is different from effective surface temperature  $T_{eff}$ . For same  $T_{eff}^\infty$ , effective surface temperature  $T_{eff}$  for higher mass would be higher which explains the trend resulting from higher  $\alpha_r$  and correspondingly higher amplitude of the strain tensor  $h_0$ . The trend will be reversed if instead of  $T_{eff}^\infty$ ,  $T_{eff}$  is chosen to be the same. In Fig.-1, the amplitude of the strain

tensor  $h_0$  has been plotted as a function of evolving angular velocity of the star  $\Omega$  in units of  $\text{rad s}^{-1}$  resulting from the GW emission for three different NS masses. On the contrary, Fig.-2, is same but  $\alpha_r$  has been estimated from ‘spin equilibrium’ using and experimental data has been taken from the source IGR J00291 of Table-2 of Ref.[6]. The results for NS mass  $1.9004 M_\odot$  is very close but slightly higher than that for mass  $1.4369 M_\odot$  which is almost indistinguishable in Fig.-2. The  $\alpha_r$  estimated from ‘spin equilibrium’ is higher than that from ‘thermal equilibrium’ which corresponds to larger  $h_0$  as can be seen in Fig.-2. The trends both in Fig.-1 & 2 may seem plausible looking at the Eq.(1) which becomes clear after noticing the  $\alpha_r$  relation either from Eq.(2) (spin equilibrium) or from Eq.(4) (thermal equilibrium). This factor can be justified further as follows: We have performed the whole calculation assuming the range of angular momentum of the star so that the CFS two-stream instability applies in order to enhance the r-mode perturbation instead of damping it as the emission of GW pumps into the perturbation. Hence, the star, rotating at a particular angular velocity, emits GW due to perturbation and loses its angular momentum corresponding to which the perturbation increases and angular velocity decreases. Consequently, the intensity of gravitational emission increases while the angular velocity of the star continues to diminish until it passes through the critical limit ( $\Omega_c$ ) of CFS two-stream instability [1] and therefore the above mentioned trend.

## Results and discussion

By setting accretion torque in Eq.(12b) of Ref.[6], to zero, the saturation value of  $\alpha_r$  can be obtained using  $\frac{d\alpha_r}{dt} = 0$  which provides very high saturation value of the amplitude  $\alpha_r \sim 1$ . This would mean extremely high luminosity which has no resemblance with the observations. Thus none of the saturation mechanisms proposed so far can saturate r-modes at low amplitudes required to explain observed luminosities. To constrain the amplitude to reasonable values non-linear dissipative terms needs to be incorporated which are yet to be determined. Therefore, in this work two different methods for constraining the r-mode amplitude,  $\alpha_r$ , from observations of LMXB NS transients have been compared. The first one, which gives larger values for  $\alpha_r$ , is based on the spin equilibrium assumption where we assume that in an outburst-quiescence cycle all the spin-up torque due to accretion during the outburst is balanced by the r-mode spin-down torque due to gravitational radiation in the whole cycle.

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The second one is based on the thermal equilibrium outlined in Ref.[7], but rather than estimating the quiescent luminosity using the r-mode amplitude obtained by imposing spin equilibrium constraint, observations of the quiescent luminosity of LMXBs have been used to constrain the amplitude of the r-mode directly.

### A. Constraints from ‘Spin Equilibrium’

Due to very high compactness neutron stars are sources of tremendous gravitational pull. As a consequence, when these stars belong to some binary system they can acquire mass from the companion. So a particle free around the star, which is possible at Keplerian angular velocity, hence having a huge angular momentum finally gets absorbed. After absorption of such particle in the star, which is rotating at a rate much slower than the Keplerian, star gains angular momentum so as to make the total angular momentum conserved. Therefore, a star which is accumulating mass at a rate  $\dot{M}$  from its binary companion gradually spins up. Thus we can use the observed spin-up rates and outburst properties to constrain the r-mode amplitude directly. Therefore we have

$$2\pi I \dot{\nu} \Delta = \frac{2J_c}{\tau_{GR}} \quad (2)$$

where  $I = MR^2 \tilde{I}$  is the moment of inertia. Right hand side of the equation is the spin down torque due to gravitational emission out of r-mode perturbation. The  $J_c$  in the equation is the canonical angular momentum of the mode given by

$$J_c = -\frac{3}{2} \tilde{J} MR^2 \Omega \alpha_r^2, \quad (3)$$

the dimensionless quantities  $\tilde{J}$  and  $\tilde{I}$  are provided in Eqs.20 & 21, respectively of Ref[3],  $\dot{\nu}$  is the spin-up rate during outburst and  $\Delta = (t_o/t_r)$  is the ratio of the outburst duration  $t_o$  to the recurrence time  $t_r$ .

### B. Constraints from ‘Thermal Equilibrium’

In a steady-state, the gravitational radiation pumps energy into the r-mode at a rate given by [6]

$$W_d = (1/3)\Omega \dot{J}_c = -2\tilde{E}/\tau_{GR} \quad (4)$$

In a thermal steady-state, all of this energy must be dissipated in the star. Some fraction  $L_\nu$  of this heat will be lost from the star due to neutrino emission and the rest  $L_\gamma$  will be radiated at the surface. It should be mentioned that the thermal steady state is not an assumption but a rigorous result when the mode is saturated, and in particular it is independent of the cooling mechanism. We further assume that all of the energy emitted from the star during quiescence is due to the r-mode dissipation inside the star which implies  $W_d = L_\nu + L_\gamma$  where  $L_\nu$  and  $L_\gamma$  are the neutrino luminosity and the thermal photon luminosity at the surface of the star, respectively.

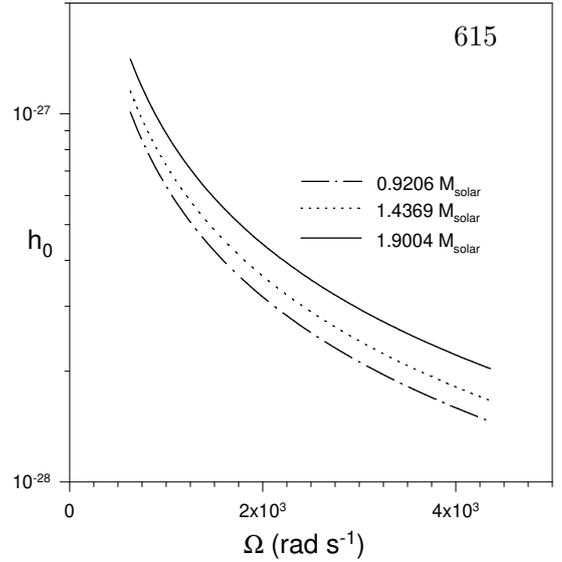


FIG. 1: A figure caption. The figure captions are automatically numbered.

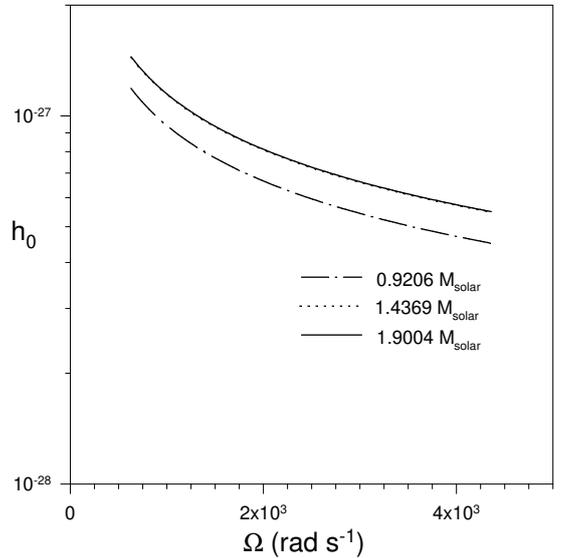


FIG. 2: A figure caption. The figure captions are automatically numbered.

## References

- [1] S. Chandrasekhar, Phys. Rev. Lett. **24**, 611 (1970).
- [2] J. L. Friedman, B. F. Schutz, Astrophys. J. **222**, 281 (1978).
- [3] S. Mukhopadhyay, J. Lahiri, D. Atta, K. Imam, D. N. Basu, Phys. Rev. C **97**, 065804(2018).
- [4] B. J. Owen, Class. Quantum Grav. **26**, 204014 (2009).
- [5] B. J. Owen, Phys.Rev. D **82**, 104002 (2010).
- [6] S. Mahmoodifar, T. Strohmayer, Astrophys. J. **773**, 140 (2013).
- [7] E. F. Brown, G. Ushomirsky, Astrophys. J. **536**, 915 (2000).